IcoAtmosBenchmark
NICAM kernels

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Contents

Chapter 1

Brief introduction of NICAM

1.1 NICAM is

NICAM (Nonhydrostatic Atmospheric ICosahedral Model) is an global atmospheric model for the climate study, mainly developed by Japan Agency for Marine-Earth Science and Technology (JAMSTEC), Atmosphere And Ocean Research Institute (AORI) of U-Tokyo, and RIKEN. Since aiming high-resolution global atmospheric simulation, this model has several features, such as;

- cloud-system resolving approach,
- icosahedral grid system,
- targetting massively parallel supercomputer.

This manual describes the overview of *NICAM* and each kernel program briefly. For the details of *NICAM*, see Tomita and Satoh (2004), Satoh et al. (2008), Satoh et al. (2014), etc.

1.2 Governing equations and the coordinate system

The governing equations of *NICAM* are based on a fully compressible non-hydrostatic system, including acoustic waves, fast gravity waves and slow gravity waves. As a coordinate system, *NICAM* uses the Cartesian coordinate in 3-dimensional space. Introducing an orthogonal basis $\{\boldsymbol{e_1}, \boldsymbol{e_2}, \boldsymbol{e_3}\}$, that is independent of space with $\boldsymbol{e_3}$ being in the same direction as the angular velocity of the Earth, a 3-dimensional velocity $\boldsymbol{v}$ on the Earth's surface with reference to the bases $\boldsymbol{e_1}$, $\boldsymbol{e_2}$ and $\boldsymbol{e_3}$ has the three components $\{v_1, v_2, v_3\}$. Furthermore, the velocity $\boldsymbol{v}$ is sometimes separated into a "horizontal element" and "vertical element" of that vector quantity. This treatment is useful in the horizontal-explicit-vertical-implicit scheme, which is used in NICAM. The wind component of radial direction is separated from $\boldsymbol{v}$ and named as a vertical wind w. The residual is horizontal wind and expressed as $\boldsymbol{v}_h = \{v_x, v_y, v_z\}$.

The 3-dimensional Cartesian notations are used mainly in dynamical core of NICAM. In the physics package, more general wind velocity notations are used: the zonal wind u, meridional wind v, and vertical wind w. the same velocity $\boldsymbol{v}$ can be denoted as

$$\boldsymbol{v} = u\hat{\boldsymbol{\imath}} + v\hat{\boldsymbol{\jmath}} + w\hat{\boldsymbol{k}} \tag{1.1}$$

where $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, and $\hat{\boldsymbol{k}}$ are unit vectors in the longitudinal, latitudinal direction, and outward unit vector in the vertical direction at the given point on the sphere, respectively.

NICAM have a switch for the shallow-atmosphere approximation. The shallow-atmosphere approximation has inconsistency in the conservation of absolute angular momentum without several metric terms and the vertical Coriolis term. *NICAM* introduces a "deep" factor

$$\gamma \equiv r/a \tag{1.2}$$

where r is the distance from the center of the Planet, and a is the radius of the Planet at sea level. If we use the shallow-atmosphere approximation, γ is set to 1.

For vertical coodinate, *NICAM* employs a terrain-following coodinate with the following metrics:

$$\xi = \frac{\xi_T(z - z_s)}{z_T - z_s}, \qquad G^{1/2} \equiv (\frac{\partial z}{\partial \xi})_{\mathrm{h}}, \qquad \boldsymbol{G^z} \equiv \nabla_{\mathrm{h0}}\xi = -\frac{\tilde{\nabla}_{\mathrm{h0}} z}{G^{1/2}} \tag{1.3}$$

where z_T is the top of the model domain, z_s is the surface height, $(\partial/\partial\xi)_{\mathrm{h}}$ denotes the derivative along the vertical direction, and $\tilde{\nabla}_{\mathrm{h0}}$ denotes the spherical gradient operator along a constant ξ plane at sea level. Since $G^{1/2}\gamma^2$ is the factor of volume against the surface, *NICAM* treats the prognostic variable multiplied by this factor. For example, the perturbation density is $R = (\rho - \rho_{\mathrm{ref}})G^{1/2}\gamma^2$, where subscript ref means the hydrostatic reference state, the horizontal and vertical momentum are $\boldsymbol{V}_h = \rho G^{1/2}\gamma^2 \boldsymbol{v}_h$ and $W = \rho G^{1/2}\gamma^2 w$, respectively, the internal energy is $E = \rho G^{1/2}\gamma^2 e$.

The governing equations are as follows:

$$\frac{\partial R}{\partial t} + \tilde{\nabla}_{\mathrm{h0}} \cdot \frac{\boldsymbol{V}_h}{\gamma} + \frac{\partial}{\partial \xi}\left(\frac{\boldsymbol{V}_h}{\gamma}\cdot \boldsymbol{G^z} + \frac{W}{G^{1/2}}\right) = 0, \tag{1.4}$$

$$\frac{\partial \boldsymbol{V}_h}{\partial t} + \tilde{\nabla}_h \frac{P}{\gamma} + \frac{\partial}{\partial \xi}\left(\boldsymbol{G^z}\frac{P}{\gamma}\right) = -\tilde{\boldsymbol{A}}_h - \tilde{\boldsymbol{C}}_h, \tag{1.5}$$

$$\frac{\partial W}{\partial t} + \gamma^2 \frac{\partial}{\partial \xi}\left(\frac{P}{G^{1/2}\gamma^2}\right) + Rg = (-\tilde{A}_z - \tilde{C}_z), \tag{1.6}$$

$$\begin{aligned} &\frac{\partial E}{\partial t} + \tilde{\nabla}_{\mathrm{h0}} \cdot \left(h\frac{\boldsymbol{V}_h}{\gamma}\right) + \frac{\partial}{\partial \xi}\left[h\left(\frac{\tilde{\boldsymbol{V}}_h}{\gamma}\cdot \boldsymbol{G^z} + \frac{W}{G^{1/2}}\right)\right] \\ &- \left[\boldsymbol{v}_h \cdot \left(\tilde{\nabla}_{\mathrm{h0}}\frac{P}{\gamma} + \frac{\partial}{\partial \xi}\left(\boldsymbol{G^z}\frac{P}{\gamma}\right)\right) + w\left(\gamma^2\frac{\partial}{\partial \xi}\left(\frac{P}{G^{1/2}\gamma^2}\right) + Rg\right)\right] + Wg = \tilde{Q}_{heat}, \end{aligned} \tag{1.7}$$

where $P = (p - p_{\mathrm{ref}})G^{1/2}\gamma^2$ is the perturbation pressure, h is the enthalpy, g is the gravitational acceleration, and $\tilde{Q}_{\mathrm{heat}}$ is the heating rate. $\tilde{\boldsymbol{A}}(= \tilde{\boldsymbol{A}}_h + \tilde{A}_z\boldsymbol{k})$ and $\tilde{\boldsymbol{C}}(= \tilde{\boldsymbol{C}}_h + \tilde{C}_z\boldsymbol{k})$ are momentum advection term and Coriolis term, respectively. Using an orthogonal basis $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3, \}$, which is independent of space with $\boldsymbol{e}_3$ being in the same direction as the angular velocity of the Planet $(0, 0, \Omega)$. Similarly, the three-dimensional velocity $\boldsymbol{v}$ is shown as (v_1, v_2, v_3) with regard to the basis $\boldsymbol{e_1}$, $\boldsymbol{e_2}$ and $\boldsymbol{e_3}$, respectively. Using these $\tilde{\boldsymbol{A}}$ and $\tilde{\boldsymbol{C}}$ can be expressed as

$$\tilde{\boldsymbol{A}} \equiv \sum_{i=1}^{3}\left[\tilde{\nabla}_{\mathrm{h0}} \cdot \left(v_i\frac{\boldsymbol{V}_h}{\gamma}\right) + \frac{\partial}{\partial \xi}\left[v_i\left(\frac{\boldsymbol{V}_h}{\gamma}\cdot \boldsymbol{G^z} + \frac{W}{G^{1/2}}\right)\right]\right]\boldsymbol{e}_i, \tag{1.8}$$

$$\tilde{\boldsymbol{C}} \equiv 2\Omega\rho G^{1/2}\gamma^2(-v_2\boldsymbol{e}_1 + v_1\boldsymbol{e}_2). \tag{1.9}$$

1.3 Time integration

The set of governing equations that *NICAM* uses is an elastic system, so it contains all the waves, especially acoustic waves, high-frequency gravity waves and the Lamb waves. These are called as the “fast modes”. The governing equations can be described schematically as

$$\frac{\partial \Psi}{\partial t} - F = S, \tag{1.10}$$

where Ψ, F and S represent the prognostic variable, the fast-mode term, and the slow-mode term, respectively. For small time step integration a forward-backward scheme based on the HEVI (horizontally explicit and vertically implicit) method is used, while for large time step integration the second- or third-stage Runge-Kutta method (Wicker and Skamarock, 2002) is used. This scheme treats fast waves implicitly in the vertical direction only, and Helmholtz equation is one-dimensional and can be solved directly as by the tridiagonal matrix solver (See section 1.6). The time step is restricted by the horizontal grid space, explicit scheme is

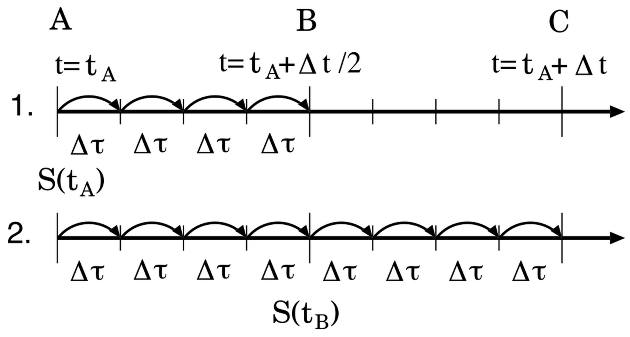


Figure 1.1: Schematic figure of temporal integration.

used in that direction. The fast-mode term are evaluated at every small time step $\Delta\tau$, and the slow-mode term are evaluatied at larger time step Δt (Figure 1.1).

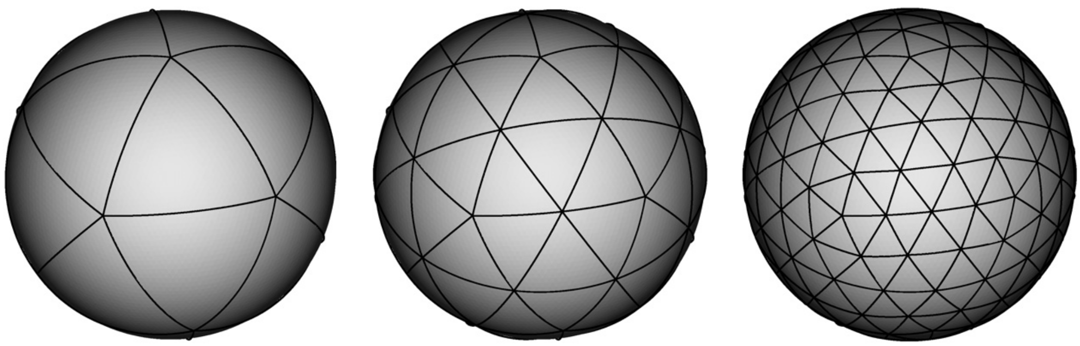
If Ψ at $t = t_A$ is given, the slow-mode tendency $S(t_A)$ can be evaluated. The variable is integrated from t_A to $t_B(> t_A)$ using $S(t_A)+F(t_A+m\Delta\tau)$ as the forcing function at $t = t_A+m\Delta\tau$, where $F(t_A+m\Delta\tau)$ is the fast-mode tendency being updated at every small step and m is the index of small time steps. Repeating integration with this small step, the temporary value of the prognostic variable Ψ^* at t_B can be obtained. The slow-mode tendency $S(t_B)$ is evaluated using this value. Finally, integrated variable Ψ at $t = t_C(> t_B)$ is calcurated by $S(t_B) + F(t_A + m\Delta\tau)$.

1.4 Icosahedral grid and "glevel"

To overcome the pole problem which old-fashioned global models have, *NICAM* uses the icosahedral grid system. In this grid system, grid points are distributed quasi-homogeneously on the sphere. The icosahedral grid can be constructed by a recursive division method which is similar to that of Stuhne and Peltier (1996). The grid resolution obtained by i-th dividing operation is called "glevel-i". First, vertices of the spherical icosahedron construct glevel-0 (Figure 1.2a). By connecting the midpoint of each edge of the triangles, the next finer grid glevel-1 is generated as Figure 1.2b. Doing the same procedure, glevel-2 is generated(Figure 1.2c). For example, Figure 1.3 shows the glevel-5 grid.

All the variables are defined at the vertices of triangular grid elements. This arrangement is categorized into the Arakawa-A type grid. The control volume is defined as the polygon constructed by connecting the gravitational centers of neighboring triangular grid elements. The shape of control volume is hexagon, as shown in Figure 1.4, except that it is pentagon at only twelve points inherited from the original icosahedron.

NICAM adopts the finite volume method for the discretization of differential operators. In Figure 1.4,



(a) glevel=0 (b) glevel=1 (c) glevel=2

Figure 1.2: The method of horizontal grid refinement.

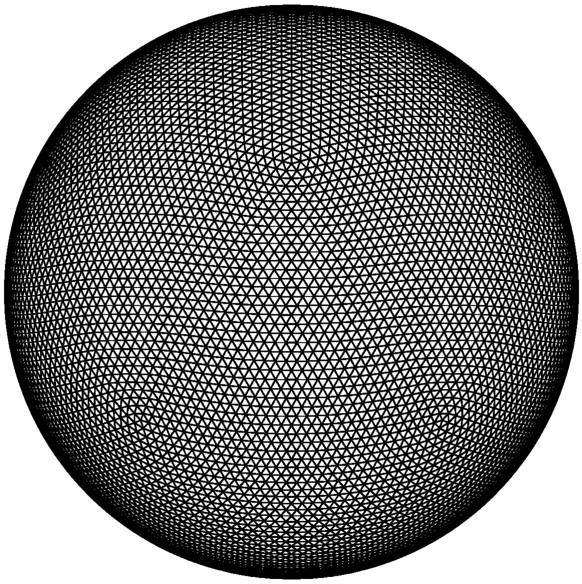


Figure 1.3: The icosahedral grid with glevel-5

some vector $\boldsymbol{u}$ at the vertices of control volume Q_i are calculated by interpolation as[*1)]

$$\boldsymbol{u}(Q_i) \simeq \frac{\alpha \boldsymbol{u}(P_0) + \beta \boldsymbol{u}(P_i) + \gamma \boldsymbol{u}(P_{i+1})}{\alpha + \beta + \gamma}, \tag{1.11}$$

where α, β and γ are the areas of the triangle $Q_i P_i P_{i+1}$, $Q_0 P_{i+1} P_0$ and $Q_i P_0 P_i$, respectively. The number 6 is replaced with 5 at the pentagonal control volumes for the singular points.

The divergence is calculated from the Gauss theorem as

$$\nabla \cdot \boldsymbol{u}(P_0) \simeq \frac{1}{a(P_0)} \sum_{i=1}^{6} b_i \frac{\boldsymbol{u}(Q_i) + \boldsymbol{u}(Q_{i+1})}{2} \cdot \boldsymbol{n}_i, \tag{1.12}$$

where b_i and $\boldsymbol{n}_i$ denote the geodesic arc length of $Q_i Q_{i+1}$ and the outward unit vector normal to this arc at the midpoint of $Q_i Q_{i+1}$. $a(P_0)$ is the area of control volume at the point P_0. More than that, Equation 1.12 can be rewritten as a linear combination:

$$\nabla \cdot \boldsymbol{u}(P_0) = \sum_{i=1}^{6} \boldsymbol{c}_i \cdot \boldsymbol{u}(P_i) \tag{1.13}$$

where $\boldsymbol{c}_i$ are constant vector coefficients which can be pre-calculated once and stored for subsequent use. In Equation 1.13, the number of addition and multiplication is the same, this formulation is preferable for the CPU which has FMA.

The gradient operator to an arbitrary variable ϕ is calculated as

$$\nabla \phi(P_0) \simeq \frac{1}{a(P_0)} \sum_{i=1}^{6} b_i \frac{\phi(Q_i) + \phi(Q_{i+1})}{2} \boldsymbol{n}_i - \frac{\phi_0}{a(P_0)} \sum_{i=1}^{6} b_i \boldsymbol{n}_i, \tag{1.14}$$

where $\phi(Q_i)$ is interpolated by the similar way to the Equation 1.11. This scheme given as Equation 1.14 generates the vertical component because the allocation points are on the sphere. In practice, the vertical component is removed after the operation of Equation 1.14.

More precisely, *NICAM* used modified icosahedoral grid by using spring dynamics in order to avoid severe problems on the numerical accuracy and stability. See Tomita et al. (2001) for the details.

[*1)] P_{i+1} should be $P_{1+\mathrm{mod}(i,6)}$, and the same hereinafter.

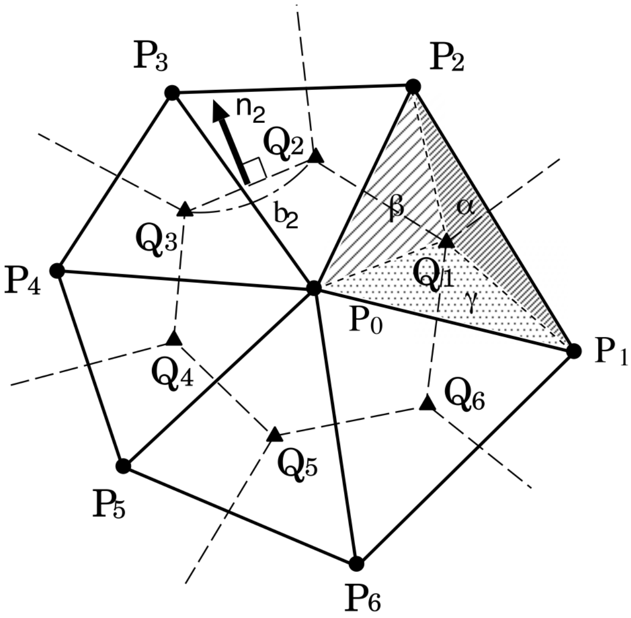


Figure 1.4: Schematic figure of control volume. (Tomita et al., 2008)

1.5 Upwind-Biased Advection Scheme

NICAM uses an upwind-biased advection scheme for icosahedral grid derived by Miura (2007). Figure 1.5 (a) shows arrangement of distorted hexagonal cells. Computational nodes are $\boldsymbol{P}_i (i = 0, 2, ..., 6)$, hexagonal cell corners are $\boldsymbol{Q}_i$, center of hexagonal cell faces are $\boldsymbol{R}_i$, lengths of hexagonal cell faces are l_i, and unit vectors normal to cell faces are $\boldsymbol{n}_i$. Figure 1.5 (b) shows schematic of the area swept by the arc $\boldsymbol{Q}'_i\boldsymbol{Q}'_{i+1}$ during a time interval. Here we assumes that the arc $\boldsymbol{Q}'_i\boldsymbol{Q}'_{i+1}$ at time t moves with a constant velocity $\boldsymbol{v}_{\boldsymbol{R}}^{t+\Delta t/2}$ and concides with the arc $\boldsymbol{Q}_i\boldsymbol{Q}_{i+1}$ at time $t + \Delta t$. The total amount of flux through the edge $\boldsymbol{Q}_i\boldsymbol{Q}_{i+1}$ during the time interval Δt is approximated by the amount of a tracer inside the parallelogram $\boldsymbol{Q}'_i\boldsymbol{Q}'_{i+1}\boldsymbol{Q}_{i+1}\boldsymbol{Q}_i$, Profiles of ρ and q, he density and the tracer mixing ratio, respectively, inside this parallelogram can be approximated by two-dimensional linear surface. Then the amount of a tracer inside this parallelogram is

$$\rho_{\boldsymbol{R}_i} q_{\boldsymbol{R}_i} = \frac{\int_{S_i} \rho q \, \mathrm{d}S}{S_i} = \rho_{\boldsymbol{C}_i} q_{\boldsymbol{C}_i}, \tag{1.15}$$

where S_i and $\boldsymbol{C}_i$ denotes the area and the mass centroid of this parallelogram, respectively. Similarly, the continuity equation will yield

$$\rho_{\boldsymbol{R}_i} = \rho_{\boldsymbol{C}_i}. \tag{1.16}$$

Thus,

$$q_{\boldsymbol{R}_i} = q_{\boldsymbol{C}_i}. \tag{1.17}$$

The position of $\boldsymbol{C}_i$ can be computed as

$$\boldsymbol{C}_i = \boldsymbol{R}_i - v_{\boldsymbol{R}_i}^{t+\Delta t/2} \frac{\Delta t}{2}. \tag{1.18}$$

Linear surfaces inside the parallelogram can be approximated by nodal values and gradients at a computational node that shares the cell face and is on the upwind side of the cell face. See Miura (2007) for more details.

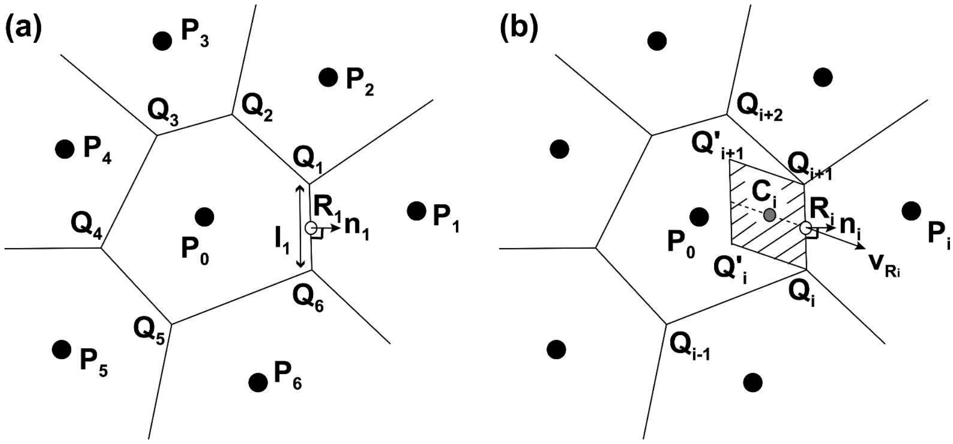


Figure 1.5: (a) Arrangement and notation of distorted hexagonal cells.(b) Schematic of the area swept by the arc $\boldsymbol{Q}'_i\boldsymbol{Q}'_{i+1}$ during a time interval.

1.6 Vertical coordinate and discretization

NICAM uses the Lorenz grid for the vertical grid configuration as shown in Figure 1.6, where W (vertical velocity with metrics) is defined at the half-integer levels and the other variables are defined at the integer levels. This model allows the grid stretching in the ξ coordinate as show in Figure 1.6. The integer levels are located at the mid-point of the upper and lower half-integer levels. The linear interpolation is used to obtain the values at the half-integer levels from the values at the integer levels, and vice versa:

$$\phi_{k+1/2} = a_{k+1/2}\,\phi_{k+1} + b_{k+1}\,\phi_k, \tag{1.19}$$

$$\phi_k = c_k\,\phi_{k+1/2} + d_k\,\phi_{k-1/2}, \tag{1.20}$$

where

$$a_{k+1/2} = \frac{\xi_{k+1/2} - \xi_k}{\xi_{k+1} - \xi_k}, \tag{1.21}$$

$$b_{k+1/2} = 1 - a_{k+1/2}, \tag{1.22}$$

$$c_k = \frac{\xi_k - \xi_{k-1/2}}{\xi_{k+1/2} - \xi_{k-1/2}}, \tag{1.23}$$

$$d_k = 1 - c_k. \tag{1.24}$$

(1.25)

One of main solvers is that of the Helmholtz equation, which can be expressed as

$$-\frac{\partial}{\partial\xi}\left[A^o\frac{\partial(A^i\tilde{W})}{\partial\xi}\right] - \frac{\partial(B\tilde{W})}{\partial\xi} - C^o\frac{\partial C^i\tilde{W}}{\partial\xi} + \alpha D\tilde{W} = S^{\mathrm{all}} \tag{1.26}$$

where

$$\tilde{W} \equiv \frac{W^{*\tau+\Delta\tau}}{G^{1/2}\gamma^2}\left(= (\rho w)^{*\tau+\Delta\tau}\right) \tag{1.27}$$

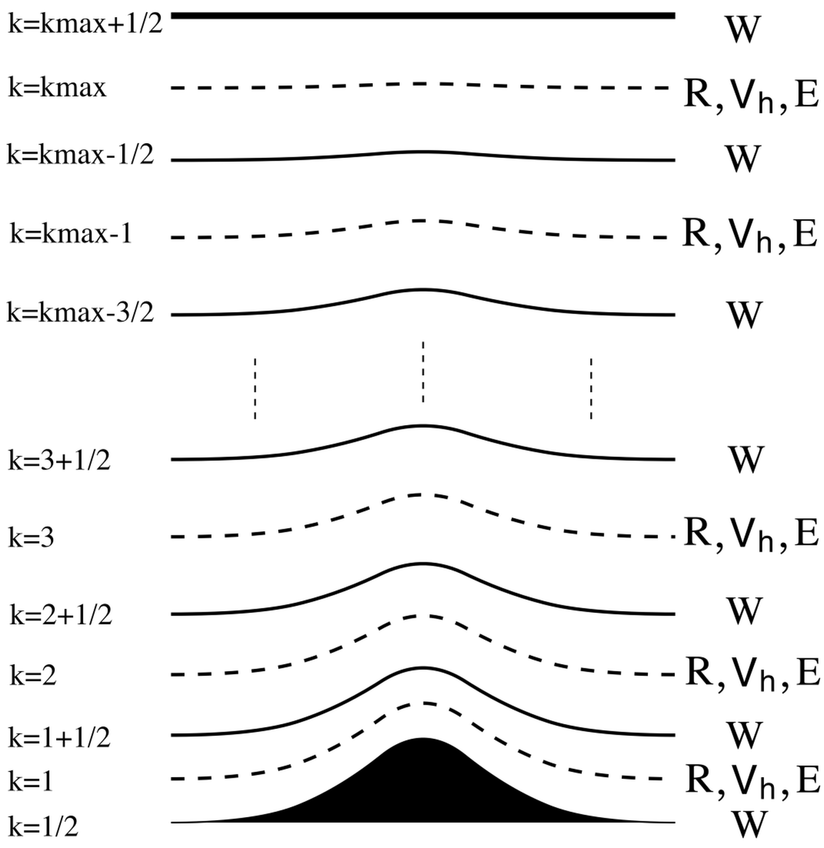


Figure 1.6: Schematic figure of vertical grid configuration.

and the coefficients are given as

$$A^o = \frac{1}{G^{1/2}\gamma^2}, \tag{1.28}$$

$$A^i = \gamma^2 h^t, \tag{1.29}$$

$$B = \tilde{g}^t, \tag{1.30}$$

$$C^o = \frac{C_v g}{R_d \gamma^2}, \tag{1.31}$$

$$C^i = \gamma^2, \tag{1.32}$$

$$D = \frac{C_v G^{1/2}}{R_d \Delta\tau^2}. \tag{1.33}$$

The source term is given as

$$S^{\text{all}} = \frac{C_v}{R_d \Delta\tau^2}\left(\frac{\alpha W^{*\tau} + \Delta\tau S^W}{\gamma^2} - \Delta\tau \frac{\partial}{\partial \xi}\left[\frac{1}{G^{1/2}\gamma^2}(P^{*\tau} + \Delta\tau S^P)\right] - \frac{\Delta\tau}{\gamma^2}(R^{*\tau} + \Delta\tau S^R) g\right). \tag{1.34}$$

Equation 1.26 is in discretized form at the level $k + 1/2$:

$$
\begin{aligned}
&-\left[\frac{A^o_{k+1}\left(A^i_{k+3/2}\tilde{W}_{k+3/2}-A^i_{k+1/2}\tilde{W}_{k+1/2}\right)}{\Delta\xi_{k+1}\Delta\xi_{k+1/2}}-\frac{A^o_{k}\left(A^i_{k+1/2}\tilde{W}_{k+1/2}-A^i_{k-1/2}\tilde{W}_{k-1/2}\right)}{\Delta\xi_{k}\Delta_{k+1/2}}\right]\\
&-\left[\frac{\left(c_{k+1}B_{k+3/2}\tilde{W}_{k+3/2}+d_{k+1}B_{k+1/2}\tilde{W}_{k+1/2}\right)-\left(c_{k}B_{k+1/2}\tilde{W}_{k+1/2}+d_{k}B_{k-1/2}\tilde{W}_{k-1/2}\right)}{\Delta\xi_{k+1/2}}\right]\\
&-C^o_{k+1/2}\left[\frac{\left(c_{k+1}C^i_{k+3/2}\tilde{W}_{k+3/2}+d_{k+1}C^i_{k+1/2}\tilde{W}_{k+1/2}\right)-\left(c_{k}C^i_{k+1/2}\tilde{W}_{k+1/2}+d_{k}C^i_{k-1/2}\tilde{W}_{k-1/2}\right)}{\Delta\xi_{k+1/2}}\right]\\
&+\alpha D_{k+1/2}\tilde{W}_{k+1/2}\\
&=S^{\text{all}}_{k+1/2} \qquad (1.35)
\end{aligned}
$$

where

$$
\begin{aligned}
S^{\text{all}}_{k+1/2}=&\frac{C_v}{R_d\Delta\tau^2}\left[\left(\frac{\alpha W^{*\tau}_{k+1/2}+\Delta\tau S^W_{k+1/2}}{\gamma^2_{k+1/2}}\right)\right.\\
&-\frac{\Delta\tau}{\Delta\xi_{k+1/2}}\left(\frac{P^{*\tau}_{k+1}+\Delta\tau S^P_{k+1}}{G^{1/2}_{k+1}\gamma_{k+1}{}^2}-\frac{P^{*\tau}_{k}+\Delta\tau S^P_{k}}{G^{1/2}_{k}\gamma^2_{k}}\right)\\
&\left.-g\Delta\tau\left(a_{k+1/2}\frac{R^{*\tau}_{k+1}+\Delta\tau S^R_{k+1}}{\gamma^2_{k+1}}+b_{k+1/2}\frac{R^{*\tau}_{k}+\Delta\tau S^R_{k}}{\gamma^2_{k}}\right)\right] \qquad (1.36)
\end{aligned}
$$

Equation 1.35 is a tridiagonal matrix system as follows:

$$
\begin{aligned}
M^L_{k+1/2}\tilde{W}_{k-1/2}+M^C_{k+1/2}\tilde{W}_{k+1/2}+M^U_{k+1/2}\tilde{W}_{k+3/2}=S^{\text{all}}_{k+1/2},\\
\text{for}\quad \frac{3}{2}\le k+\frac{1}{2}\le k_{\text{max}}-\frac{1}{2},
\end{aligned} \qquad (1.37)
$$

where

$$
\begin{aligned}
M^C_{k+1/2}=&\quad\alpha D_{k+1/2}\\
&+\frac{1}{\Delta\xi_{k+1/2}}\left[\frac{A^o_{k+1}A^i_{k+1/2}}{\Delta\xi_{k+1}}+\frac{A^o_{k}A^i_{k+1/2}}{\Delta\xi_{k}}-(d_{k+1}-c_k)(B_{k+1/2}+C^o_{k+1/2}C^i_{k+1/2})\right], \qquad (1.38)\\
M^U_{k+1/2}=&-\frac{A^o_{k+1}A^i_{k+3/2}}{\Delta\xi_{k+1}\Delta\xi_{k+1/2}}-\frac{c_{k+1}(B_{k+3/2}+C^o_{k+1/2}C^i_{k+3/2})}{\Delta\xi_{k+1/2}}, \qquad (1.39)\\
M^L_{k+1/2}=&-\frac{A^o_{k+1}A^i_{k-1/2}}{\Delta\xi_{k+1}\Delta\xi_{k+1/2}}+\frac{d_{k}(B_{k-1/2}+C^o_{k+1/2}C^i_{k-1/2})}{\Delta\xi_{k+1/2}}. \qquad (1.40)
\end{aligned}
$$

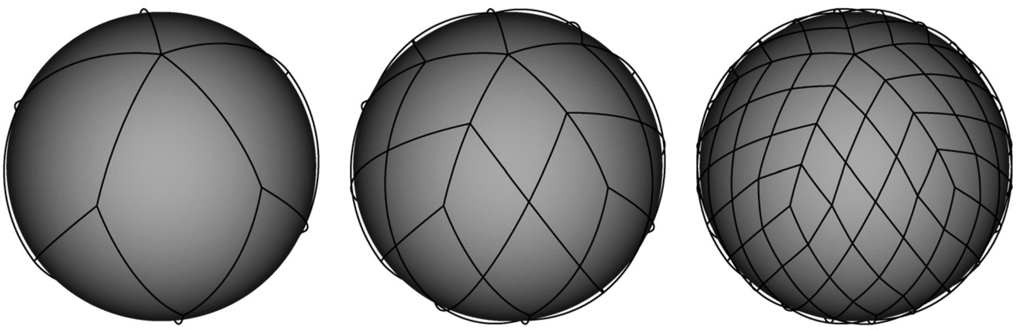
If the boundary conditions for $\tilde{W}$ are given at $k=1-1/2$ and $k_{\text{max}}+1/2$, this linear system can be written explicitly as

$$
\begin{pmatrix}
M^C_{1+1/2} & M^U_{1+1/2} & 0 & \cdots & \cdots & \cdots & 0 \\
M^L_{2+1/2} & M^C_{2+1/2} & M^U_{2+1/2} & 0 & \cdots & \cdots & \vdots \\
0 & M^L_{3+1/2} & M^C_{3+1/2} & M^U_{3+1/2} & 0 & \cdots & \vdots \\
\vdots & & \cdots & \cdots & \cdots & & 0 \\
\vdots & \cdots & \cdots & 0 & M^L_{k_{\max}-3/2} & M^C_{k_{\max}-3/2} & M^U_{k_{\max}-3/2} \\
0 & \cdots & \cdots & \cdots & 0 & M^L_{k_{\max}-1/2} & M^C_{k_{\max}-1/2}
\end{pmatrix}
\times
\begin{pmatrix}
\tilde{W}_{1+1/2} \\ \tilde{W}_{2+1/2} \\ \tilde{W}_{3+1/2} \\ \vdots \\ \tilde{W}_{k_{\max}-3/2} \\ \tilde{W}_{k_{\max}-1/2}
\end{pmatrix}
=
\begin{pmatrix}
S^{\mathrm{all}}_{1+1/2} - M^L_{1+1/2}\tilde{W}_{1-1/2} \\
S^{\mathrm{all}}_{2+1/2} \\
S^{\mathrm{all}}_{3+1/2} \\
\vdots \\
S^{\mathrm{all}}_{k_{\max}-3/2} \\
S^{\mathrm{all}}_{k_{\max}-1/2} - M^U_{k_{\max}-1/2}\tilde{W}_{k_{\max}+1/2}
\end{pmatrix}
\tag{1.41}
$$

Since A^i and B depend on large step values(see section 1.3), the compositions of the matrix (Equation 1.38, 1.39, 1.40, 1.41) are the updated only at the large step.

1.7 Domain decomposition and "rlevel"

For domain decomposition for parallel computing, *NICAM* adopt the concept of a region-division level, or "rlevel", along with the grid-division level (Tomita et al., 2008). The ten rectangles in Figure 1.7a are the result of connecting two neighboring triangles of a spherical icosahedron. This structure is called "rlevel-0". For each rectangle, four subrectangles are generated by connecting the diagonal midgridpoints (Figure 1.7b). This structure is called "rlevel-1". This process is repeated until the desired number of regions is obtained.



(a) rlevel=0 (b) rlevel=1 (c) rlevel=2

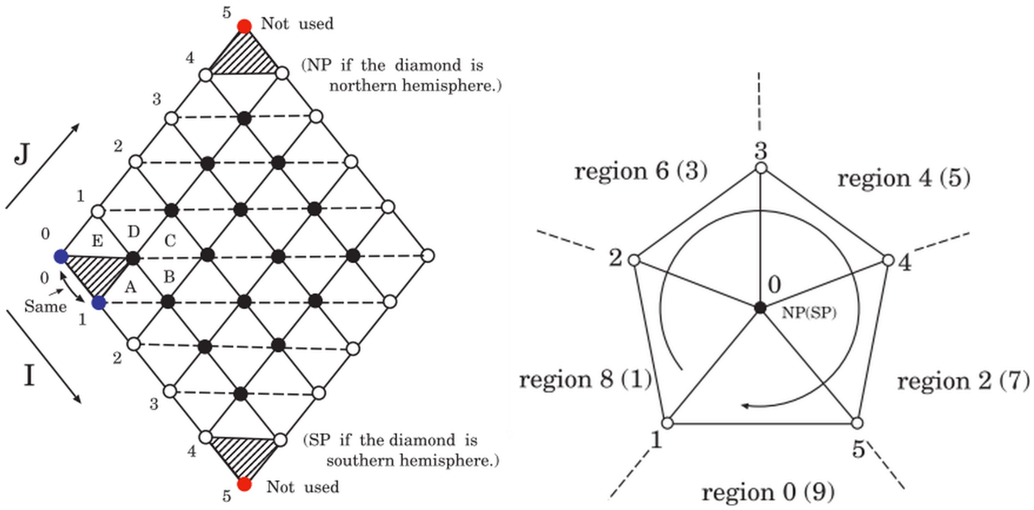
Figure 1.7: The method of region division. (Tomita et al., 2008)

1.8 Horizontal data structure

As described in section 1.4, the grid elements of an icosahedral grid are not rectangles. In *NICAM*, the shape of the control volume is hexagonal, except for the 12 points associated with the original icosahedron vertices. Therefore, the icosahedral grid is categorized as an unstructured grid. However, an icosahedral grid can actually be treated as a structured grid. An example of the grid structure in a region is shown in Figure 1.8a. The memory of each region in a single layer can be stored as a continuous one- or two-dimensional array. In

Figure 1.8a, the region manages the black gridpoints. The values indicated by the white circles are used only for reference and are provided from the neighboring regions by communication or memory copy. The values at the red circles are not used. If the west vertex of a region corresponds to a vertex of the original icosahedron, the two blue points shown in Figure 1.8a have the same values. With this data configuration, the black points cover all the icosahedral grid points except for the north and south poles.

Figure 1.8b shows data storage for the pole-region data. The gridpoint value included in this region is marked by a black circle. The other values marked by white circles are reference values, provided by communication or memory copy from the neighboring regular regions. The indices of the reference points are given in clockwise order. The pole regions are managed by the master process. Although this might cause load imbalance in parallel computing, the ratio of calculation required at the poles to that required in regular regions is small, and in practice the problem is negligible.



(a) Normal region (b) Pole region

Figure 1.8: Grid structure of a region.(Tomita et al., 2008)

1.9 Parallelization

NICAM uses MPI for parallel execution. One MPI process manages one or several regions defined in previous section. For regions managed by the same process, exchanges of boundary values are done by memory copy, otherwise done by MPI communication. Figure 1.9 shows a schematic diagram of region management. In this case, rlevel is 1 and there are 40 regions, managed by 10 MPI processes. So a single process manages 4 regions, shown by the same color.

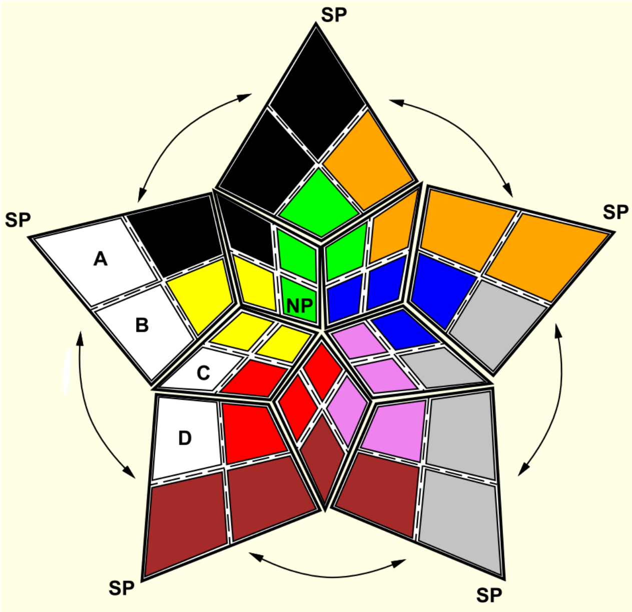


Figure 1.9: Schematic figure of parallelization. This figure shows the expansion of region geometry with rlevel-1.

1.10 Problem size

As shown in section 1.4, total number of grid points are defined by glevel. Total grid points N_g is calculated by Equation 1.42.

$$N_g = 10 \times 4^{\mathrm{gl}} + 2, \tag{1.42}$$

where gl is glevel. $+2$ is corresponding to the pole points. Table 1.1 shows total number of grid points and average grid point distance for several glevels.

Table 1.1: Number of grid points and glevel

glevel	grid points	average grid interval[m]
0	12	7 142 126
1	42	3 571 063
2	162	1 785 432
...	...	...
8	655 362	27 899
9	2 621 442	13 949
10	10 485 762	6975
11	41 943 042	3487
...	...	...
13	2 684 354 562	872

Similary, Total number of regions $N_r$is calculated by Equation 1.43.

$$N_r = 10 \times 4^{\mathrm{rl}}, \tag{1.43}$$

where rl is rlevel. Table 1.2 shows the relation of rlevel and number of regions.

So the number of grid points in a single region (except polar region) N_p is calculated by Equation 1.44.

$$N_p = (2^{\mathrm{gl}-\mathrm{rl}} + 2)^2, \tag{1.44}$$

here $+2$ in parentheses denotes the halo grid points. For example, in case glevel is 5 and rlevel is 1, N_p is 324.

Table 1.2: Number of regions and rlevel

rlevel	number of regions numbers
0	10
1	40
2	160
3	640
4	2560

Chapter 2

Description of each kernel

2.1 Overview and common stuff

There are several things you should know when you read the kernel program codes.

2.1.1 Coding rule of *NICAM*

NICAM is written with Fortran90/95 standards, and uses `module` to modularize the program. Module name begins with `mod_`, such as `mod_adm`. Almost all source file defines only one module and have the same name with the module, such as `mod_adm.f90` Several modules are used to define public parameters, variables, and subroutines, called "public object". Name of such objects are prefixed its module name. For example, `ADM_gall` is defined in module `mod_adm` that is defined in `mod_adm.f90` in original *NICAM* source. Among the public objects, variables and parameters related to the problem size are moved and defined in `problem_size.inc`, that is included in `mod_misc.f90`

2.1.2 Data array for regular region

Figure 2.1 shows the schematic figure of data layout in a regular region. The grid points managed in the region are black circles, and white and blue circles are so called "halo points", these are used only for the reference and their values are provided from the neighboring regions by the MPI communication or memory copy. If the west-most vertex of the region is the vertex of the original icosahedron, called singular points, two blue points have the same value. As you can see in Figure 2.1, all grid points can be specified by two index i and j, in the direction of southward and northword, respectively. For the computational efficiency, especially for vectorizing or using SIMD, the i and j dimension are merged, so usual variables are defined in a form shown in Table 2.1. The first dimensions corresponds to the horizontal index, the second is vertical index, and the third is region number (index) in each process.

The range of g is one from `ADM_gall`, that is the number of horizontal grid points including the halo points. Note that `ADM_gall = ADM_gall_1d*ADM_gall_1d`, where `ADM_gall_1d` is the number of grid points in i or j direction. `ADM_kall` is the actual number of vertical layers, and `ADM_lall` is the number of regions managed by a single MPI process. For example, if four regions are managed as Figure 1.9, `ADM_lall` is set to 4.

For some calculation, the values at the triangle center and those at the mid-points of arcs are required, called "the triangle point" and "the arc point", respectively. Corresponding to the one grid point, there are two triangle points and three arc points. To specify these points, one more dimension m are used. Their value are `ADM_TI`, `ADM_TJ` for the triangle point, and `ADM_AI`, `ADM_AIJ` and `ADM_AJ` for the arc point, respectively (see Figure 2.1).

2.1.3 Data array for pole region

For pole region, Figure 2.2 shows the schematic figure of data storing. Only one grid point, the pole point, is managed, and the other point marked by the white circles are the halo points, these are used only for the

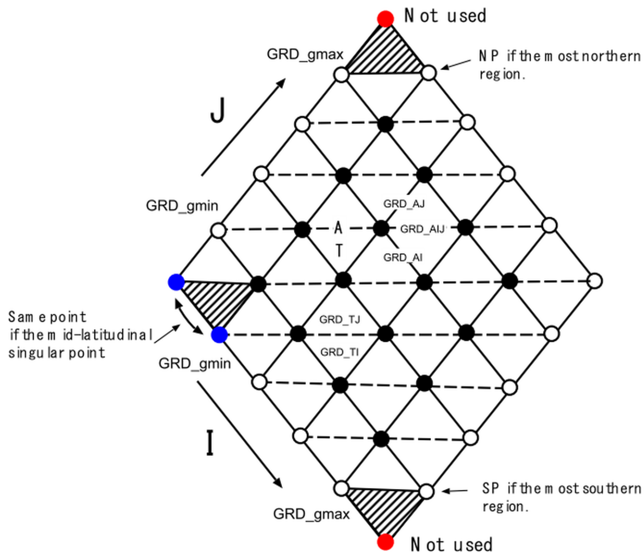


Figure 2.1: Schematic figure of data storing (regular region).

Table 2.1: Data array for the regular region.

`var(g,k,l)` : Grid point variables			
g	$=$	$1\ldots$ `ADM_gall`	horizontal index
k	$=$	$1\ldots$ `ADM_kall`	vertical index
l	$=$	$1\ldots$ `ADM_lall`	region index
`var(g,k,l,m)` : Triangle point variables			
g	$=$	$1\ldots$ `ADM_gall`	horizontal index
k	$=$	$1\ldots$ `ADM_kall`	vertical index
l	$=$	$1\ldots$ `ADM_lall`	region index
m	$=$	`ADM_TI, ADM_TJ`	index of triangle position
`var(g,k,l,m)` : Arc point variables			
g	$=$	$1\ldots$ `ADM_gall`	first horizontal index
k	$=$	$1\ldots$ `ADM_kall`	vertical index
l	$=$	$1\ldots$ `ADM_lall`	region index
m	$=$	`ADM_AI, ADM_AIJ, ADM_AJ`	index of arc position

reference and their values are provided from the neighboring regions by the MPI communication or memory copy. The indices for these halo are in order of clockwise direction. The first dimension is horizontal suffix, and the size `ADM_GALL_PL` is set to 6, one for pole point and five for halo points. The second dimension is vertical suffix, which is the same with the regular region. The third dimension is region, suffix, and the range is 1 to `ADM_LALL_PL`, which is the number of pole regions, and set to 2 (North pole and South pole).

Different from the regular region, the number of triangle points and arc points are the same with the halo points, these are no need for more dimension, and stored as the same dimensions with the grid points. But the range of index are from `ADM_GMIN_PL`(=2) to `ADM_GMAX_PL`(=6).

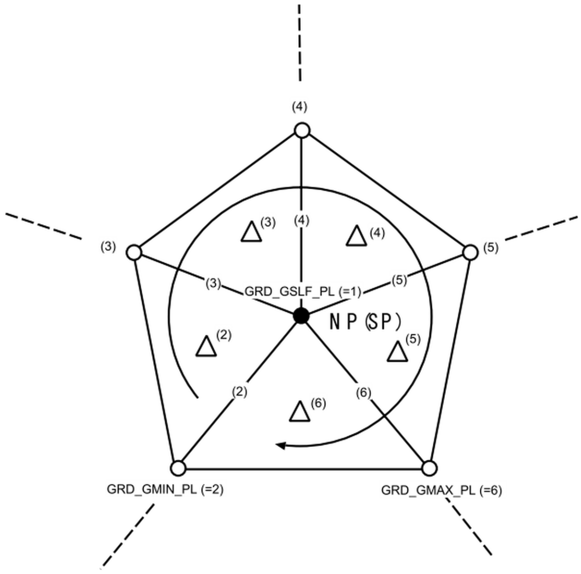


Figure 2.2: Schematic figure of data storing (pole region).

Table 2.2: Data array for the pole region.

`var_pl(n,k,l)` : Grid, triangle, arc point variables			
n	=	1 . . . `ADM_GALL_PL`	horizontal index
k	=	1 . . . `ADM_kall`	vertical index
l	=	1 . . . `ADM_LALL_PL`	region index

2.1.4 Kernelize

The kernels are single or multiple subroutines in original *NICAM* source code, and extracted and imposed into the wrapper for the kernel program. Values of input variables in the argument list of the kernel subroutine are stored as a data file just before the call in execution of the original *NICAM*, and they are read and given to the kernel subroutine in the kernel program. Similarly values of output variables in the argument list are stored just after the call in execution, and they are read and compared to the actual output values of kernel subroutine, the difference are written to the standard output for validation.

NICAM uses several "public object" defined in several modules, briefly described later. Some of them are moved to `problem_size.inc` and `includeed` in the `mod_misc`.

Kernel programs output several messages to the standard output, such as:

- min/max/sum of input data,
- min/max/sum of output data,
- min/max/sum of difference between output and validation data,

- computational time (elapsed time).

Elapsed time is measured using `omp_get_wtime()`.

There are sample output file for the reference in `reference/` directory of each kernel program, and also they are shown in"Input data and result" section of each kernel program in this document.

2.1.5 Problem size

In this kernel program package, problem size are set in `problem_size.inc` in each kernel program, except `communication`, as follows. Number of grid point of one side of the region `ADM_gall_1d` is 130 including two halo points, and the total number of grid points in whole region `ADM_gall` is 16900, which corresponds to glevel − rlevel = 7. Number of vertical layers `ADM_vlayer` is 40 and the actual size of k-direction `ADM_kall` is 42. The kernel program runs on a single process and this process manages only one normal region, ie. `ADM_lall` is 1, and also have two pole regions, ie. `ADM_have_pl` is `.true.` and `ADM_lall_pl` is 2. This normal region is to manage singular point, then `ADM_have_sgp(1)` is `.true.`

2.1.6 MPI and OpenMP

While original *NICAM* is parallelized by MPI and OpenMP, all kernel program in this package except `communication` are meant to be executed as one process and no threading. And you don't need MPI library to compile/execute these kernel programs, but you need to make OpneMP enable in order to use `omp_get_wtime()`.

2.1.7 Mesuring environment

In the following sections, the example of performance result part of the log output file of each kernel program is shown. These were measured on the machine environment shown in Table 2.3, with setting `export IAB_SYS=Ubuntu-gnu-ompi` on compilation (See `QuickStart.md`).

Table 2.3: Measuring environment

component	specification	notes
CPU	Xeon E5-2630v4 @2.2GHz (10cores) x2	HT disabled, TB enabled
Memory	256GB	
Storage	SSD (SATA)	
OS	Ubuntu 16.04.4 LTS	
Compiler	GNU 5.4.0	
MPI	OpenMPI 1.10.2	Ubuntu standard

2.2 `dyn_diffusion`

2.2.1 Description

Kernel `dyn_diffusion` is taken from the original subroutine `OPRT_diffusion` in *NICAM*. This subroutine is originally defined, as you can see in that name, in module `mod_oprt`. This module defines several differential operators on the sphere, such as divergence, gradient, etc. Subroutine `OPRT_diffusion` calculates diffusion term of given scalar field.

2.2.2 Discretization and code

Argument lists and local variables definition part of this subroutine is as follows.

```
subroutine OPRT_diffusion( &
     dscl,      dscl_pl,      &
     scl,       scl_pl,       &
     kh,        kh_pl,        &
```

```
     coef_intp, coef_intp_pl, &
     coef_diff, coef_diff_pl  )
  implicit none

  real(RP), intent(out) :: dscl        (ADM_gall   ,ADM_kall,ADM_lall   )
  real(RP), intent(out) :: dscl_pl     (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)  :: scl         (ADM_gall   ,ADM_kall,ADM_lall   )
  real(RP), intent(in)  :: scl_pl      (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)  :: kh          (ADM_gall   ,ADM_kall,ADM_lall   )
  real(RP), intent(in)  :: kh_pl       (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)  :: coef_intp   (ADM_gall   ,1:3,        ADM_nxyz,TI:TJ,ADM_lall   )
  real(RP), intent(in)  :: coef_intp_pl(ADM_gall_pl,1:3,        ADM_nxyz,      ADM_lall_pl)
  real(RP), intent(in)  :: coef_diff   (ADM_gall   ,1:6,        ADM_nxyz,      ADM_lall   )
  real(RP), intent(in)  :: coef_diff_pl(            1:ADM_vlink,ADM_nxyz,      ADM_lall_pl)

  real(RP) :: vt   (ADM_gall   ,ADM_nxyz,TI:TJ)
  real(RP) :: vt_pl(ADM_gall_pl,ADM_nxyz)
  real(RP) :: kf   (1:6)

  integer  :: gmin, gmax, iall, gall, kall, lall, nxyz, gminm1

  integer  :: ij
  integer  :: ip1j, ijp1, ip1jp1
  integer  :: im1j, ijm1, im1jm1

  integer  :: g, k, l, d, n, v
  !---------------------------------------------------------------------------
```

Here `dscl`, `dscl_pl` are calculated diffusion of regular region and polar region, respectively, from some scalar field `scl`, `scl_pl` and diffusion coefficient `kh`, `kh_pl` at the triangular points. Other arguments `coef_intp`, `coef_intp_pl`, `coef_diff`, `coef_diff_pl` are various coefficients for finite difference calculation. These coefficients are calculated in advance in the subroutine `OPRT_diffusion_setup` also defined in the module (See section 2.8). These are supplied as arguments, not module variables, because of the computational optimization. The values are read from input data before executing this subroutine.

local variables `vt`, `vt_pl` are the differentiation of given scalar field at the gravitational center of triangles. Note that the range of the last dimension is `TI:TJ`. See below for details. `ADM_nxyz` is `parameter` and the value is 3, so this dimension shows spatial direction (X,Y,Z).

The first part of this subroutine is as follows.

```
  call DEBUG_rapstart('OPRT_diffusion')

  gmin = (ADM_gmin-1)*ADM_gall_1d + ADM_gmin
  gmax = (ADM_gmax-1)*ADM_gall_1d + ADM_gmax
  iall = ADM_gall_1d
  gall = ADM_gall
  kall = ADM_kall
  lall = ADM_lall
  nxyz = ADM_nxyz

  gminm1 = (ADM_gmin-1-1)*ADM_gall_1d + ADM_gmin-1

  !$omp parallel default(none),private(g,k,l,d,ij,ip1j,ip1jp1,ijp1,im1j,ijm1,im1jm1), &
  !$omp shared(ADM_have_sgp,gminm1,gmin,gmax,iall,gall,kall,lall,nxyz,dscl,scl,kh,kf,vt,coef_intp,coef_diff)
  do l = 1, lall
  do k = 1, kall

     do d = 1, nxyz
        !$omp do
        do g = gminm1, gmax
           ij     = g
           ip1j   = g + 1
           ip1jp1 = g + iall + 1
           ijp1   = g + iall

           vt(g,d,TI) = ( ( + 2.0_RP * coef_intp(g,1,d,TI,l) &
                            - 1.0_RP * coef_intp(g,2,d,TI,l) &
                            - 1.0_RP * coef_intp(g,3,d,TI,l) ) * scl(ij    ,k,l) &
                        + ( - 1.0_RP * coef_intp(g,1,d,TI,l) &
                            + 2.0_RP * coef_intp(g,2,d,TI,l) &
                            - 1.0_RP * coef_intp(g,3,d,TI,l) ) * scl(ip1j  ,k,l) &
                        + ( - 1.0_RP * coef_intp(g,1,d,TI,l) &
                            - 1.0_RP * coef_intp(g,2,d,TI,l) &
                            + 2.0_RP * coef_intp(g,3,d,TI,l) ) * scl(ip1jp1,k,l) &
                        ) / 3.0_RP
        enddo
```

```
      !$omp end do nowait

      !$omp do
      do g = gminm1, gmax
         ij     = g
         ip1j   = g + 1
         ip1jp1 = g + iall + 1
         ijp1   = g + iall

         vt(g,d,TJ) = ( ( + 2.0_RP * coef_intp(g,1,d,TJ,l) &
                          - 1.0_RP * coef_intp(g,2,d,TJ,l) &
                          - 1.0_RP * coef_intp(g,3,d,TJ,l) ) * scl(ij    ,k,l) &
                      + ( - 1.0_RP * coef_intp(g,1,d,TJ,l) &
                          + 2.0_RP * coef_intp(g,2,d,TJ,l) &
                          - 1.0_RP * coef_intp(g,3,d,TJ,l) ) * scl(ip1jp1,k,l) &
                      + ( - 1.0_RP * coef_intp(g,1,d,TJ,l) &
                          - 1.0_RP * coef_intp(g,2,d,TJ,l) &
                          + 2.0_RP * coef_intp(g,3,d,TJ,l) ) * scl(ijp1  ,k,l) &
                      ) / 3.0_RP
      enddo
      !$omp end do
   enddo

   if ( ADM_have_sgp(l) ) then ! pentagon
      !$omp master
      vt(gminm1,XDIR,TI) = vt(gminm1+1,XDIR,TJ)
      vt(gminm1,YDIR,TI) = vt(gminm1+1,YDIR,TJ)
      vt(gminm1,ZDIR,TI) = vt(gminm1+1,ZDIR,TJ)
      !$omp end master
   endif
```

In the first part, `vt(:,:,TI)`, `vt(:,:,TJ)` are calculated from `scl` with interpolation. Figure 2.3 shows grid arrangements. `scl` are defined on white circle points in the figure, and `vt` are defined on black circle points. See Figure 2.4 for grid arrangement. `vt` is defined on the black circle points in the figure, while other physical quantities are defined on the white circle points.

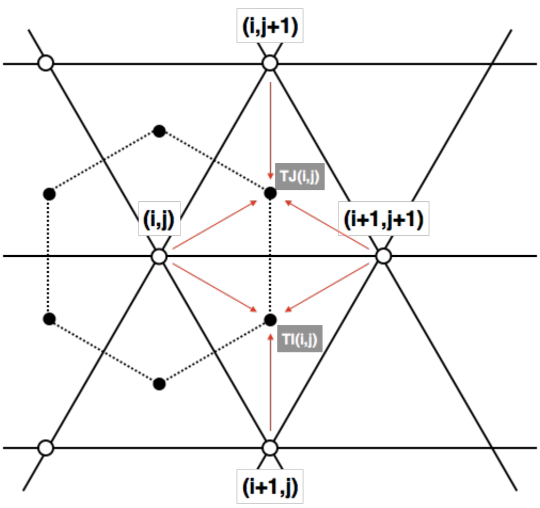


Figure 2.3: interpolation for `vt`

The second part of this subroutine is as follows.

```
!OCL XFILL
   !$omp do
   do g = 1, gmin-1
      dscl(g,k,l) = 0.0_RP
   enddo
   !$omp end do nowait

   !$omp do
   do g = gmin, gmax
      ij     = g
      ip1j   = g + 1
      ip1jp1 = g + iall + 1
```

```
         ijp1   = g + iall
         im1j   = g - 1
         im1jm1 = g - iall - 1
         ijm1   = g - iall

         kf(1) = 0.5_RP * ( kh(ij    ,k,l) + kh(ip1jp1,k,l) )
         kf(2) = 0.5_RP * ( kh(ij    ,k,l) + kh(ijp1  ,k,l) )
         kf(3) = 0.5_RP * ( kh(im1j  ,k,l) + kh(ij    ,k,l) )
         kf(4) = 0.5_RP * ( kh(im1jm1,k,l) + kh(ij    ,k,l) )
         kf(5) = 0.5_RP * ( kh(ijm1  ,k,l) + kh(ij    ,k,l) )
         kf(6) = 0.5_RP * ( kh(ij    ,k,l) + kh(ip1j  ,k,l) )

         dscl(g,k,l) = ( kf(1) * coef_diff(g,1,XDIR,l) * ( vt(ij    ,XDIR,TI) + vt(ij    ,XDIR,TJ) ) &
                       + kf(2) * coef_diff(g,2,XDIR,l) * ( vt(ij    ,XDIR,TJ) + vt(im1j  ,XDIR,TI) ) &
                       + kf(3) * coef_diff(g,3,XDIR,l) * ( vt(im1j  ,XDIR,TI) + vt(im1jm1,XDIR,TJ) ) &
                       + kf(4) * coef_diff(g,4,XDIR,l) * ( vt(im1jm1,XDIR,TJ) + vt(im1jm1,XDIR,TI) ) &
                       + kf(5) * coef_diff(g,5,XDIR,l) * ( vt(im1jm1,XDIR,TI) + vt(ijm1  ,XDIR,TJ) ) &
                       + kf(6) * coef_diff(g,6,XDIR,l) * ( vt(ijm1  ,XDIR,TJ) + vt(ij    ,XDIR,TI) ) )
      enddo
      !$omp end do

      !$omp do
      do g = gmin, gmax
         ij     = g
         ip1j   = g + 1
         ip1jp1 = g + iall + 1
         ijp1   = g + iall
         im1j   = g - 1
         im1jm1 = g - iall - 1
         ijm1   = g - iall

         kf(1) = 0.5_RP * ( kh(ij    ,k,l) + kh(ip1jp1,k,l) )
         kf(2) = 0.5_RP * ( kh(ij    ,k,l) + kh(ijp1  ,k,l) )
         kf(3) = 0.5_RP * ( kh(im1j  ,k,l) + kh(ij    ,k,l) )
         kf(4) = 0.5_RP * ( kh(im1jm1,k,l) + kh(ij    ,k,l) )
         kf(5) = 0.5_RP * ( kh(ijm1  ,k,l) + kh(ij    ,k,l) )
         kf(6) = 0.5_RP * ( kh(ij    ,k,l) + kh(ip1j  ,k,l) )

         dscl(g,k,l) = dscl(g,k,l) + ( kf(1) * coef_diff(g,1,YDIR,l) * ( vt(ij    ,XDIR,TI) + vt(ij    ,YDIR,TJ) ) &
                                     + kf(2) * coef_diff(g,2,YDIR,l) * ( vt(ij    ,XDIR,TJ) + vt(im1j  ,YDIR,TI) ) &
                                     + kf(3) * coef_diff(g,3,YDIR,l) * ( vt(im1j  ,XDIR,TI) + vt(im1jm1,YDIR,TJ) ) &
                                     + kf(4) * coef_diff(g,4,YDIR,l) * ( vt(im1jm1,XDIR,TJ) + vt(im1jm1,YDIR,TI) ) &
                                     + kf(5) * coef_diff(g,5,YDIR,l) * ( vt(im1jm1,XDIR,TI) + vt(ijm1  ,YDIR,TJ) ) &
                                     + kf(6) * coef_diff(g,6,YDIR,l) * ( vt(ijm1  ,XDIR,TJ) + vt(ij    ,YDIR,TI) ) )
      enddo
      !$omp end do

      !$omp do
      do g = gmin, gmax
         ij     = g
         ip1j   = g + 1
         ip1jp1 = g + iall + 1
         ijp1   = g + iall
         im1j   = g - 1
         im1jm1 = g - iall - 1
         ijm1   = g - iall

         kf(1) = 0.5_RP * ( kh(ij    ,k,l) + kh(ip1jp1,k,l) )
         kf(2) = 0.5_RP * ( kh(ij    ,k,l) + kh(ijp1  ,k,l) )
         kf(3) = 0.5_RP * ( kh(im1j  ,k,l) + kh(ij    ,k,l) )
         kf(4) = 0.5_RP * ( kh(im1jm1,k,l) + kh(ij    ,k,l) )
         kf(5) = 0.5_RP * ( kh(ijm1  ,k,l) + kh(ij    ,k,l) )
         kf(6) = 0.5_RP * ( kh(ij    ,k,l) + kh(ip1j  ,k,l) )

         dscl(g,k,l) = dscl(g,k,l) + ( kf(1) * coef_diff(g,1,ZDIR,l) * ( vt(ij    ,XDIR,TI) + vt(ij    ,ZDIR,TJ) ) &
                                     + kf(2) * coef_diff(g,2,ZDIR,l) * ( vt(ij    ,XDIR,TJ) + vt(im1j  ,ZDIR,TI) ) &
                                     + kf(3) * coef_diff(g,3,ZDIR,l) * ( vt(im1j  ,XDIR,TI) + vt(im1jm1,ZDIR,TJ) ) &
                                     + kf(4) * coef_diff(g,4,ZDIR,l) * ( vt(im1jm1,XDIR,TJ) + vt(im1jm1,ZDIR,TI) ) &
                                     + kf(5) * coef_diff(g,5,ZDIR,l) * ( vt(im1jm1,XDIR,TI) + vt(ijm1  ,ZDIR,TJ) ) &
                                     + kf(6) * coef_diff(g,6,ZDIR,l) * ( vt(ijm1  ,XDIR,TJ) + vt(ij    ,ZDIR,TI) ) )
      enddo
      !$omp end do nowait

!OCL XFILL
      !$omp do
      do g = gmax+1, gall
         dscl(g,k,l) = 0.0_RP
      enddo
      !$omp end do
   enddo ! loop k
   enddo ! loop l
   !$omp end parallel
```

In this part, objective variable `dscl` is calculated by interpolation of `vt`. Figure 2.4 shows the grid arrangements. `vt` are defined on black circle points as Figure 2.3, and `dscl` are defined on grey diamond points.

There are three similar do-loops, each of them calculates the contribution from X, Y, Z direction, respectively. Note that the third dimension of `coef_diff` in the loop.

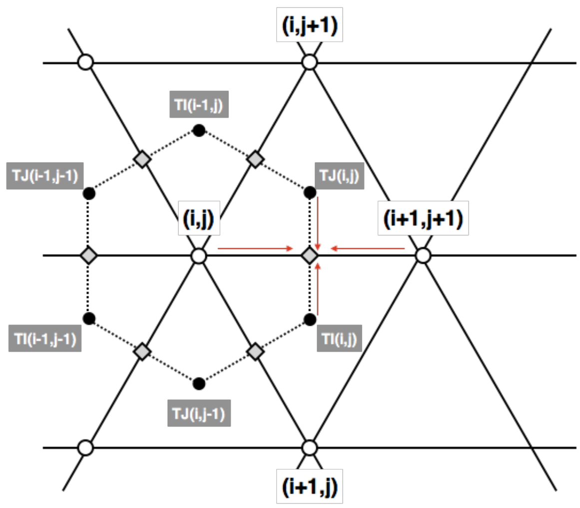


Figure 2.4: Interpolation for `dscl`

The last part of this subroutine is as follows.

```
 if ( ADM_have_pl ) then
    n = ADM_gslf_pl

    do l = 1, ADM_lall_pl
    do k = 1, ADM_kall

       do d = 1, ADM_nxyz
          do v = ADM_gmin_pl, ADM_gmax_pl
             ij   = v
             ijp1 = v + 1
             if( ijp1 == ADM_gmax_pl+1 ) ijp1 = ADM_gmin_pl

             vt_pl(ij,d) = ( ( + 2.0_RP * coef_intp_pl(v,1,d,l) &
                               - 1.0_RP * coef_intp_pl(v,2,d,l) &
                               - 1.0_RP * coef_intp_pl(v,3,d,l) ) * scl_pl(n   ,k,l) &
                           + ( - 1.0_RP * coef_intp_pl(v,1,d,l) &
                               + 2.0_RP * coef_intp_pl(v,2,d,l) &
                               - 1.0_RP * coef_intp_pl(v,3,d,l) ) * scl_pl(ij  ,k,l) &
                           + ( - 1.0_RP * coef_intp_pl(v,1,d,l) &
                               - 1.0_RP * coef_intp_pl(v,2,d,l) &
                               + 2.0_RP * coef_intp_pl(v,3,d,l) ) * scl_pl(ijp1,k,l) &
                           ) / 3.0_RP
          enddo
       enddo

       dscl_pl(:,k,l) = 0.0_RP

       do v = ADM_gmin_pl, ADM_gmax_pl
          ij   = v
          ijm1 = v - 1
          if( ijm1 == ADM_gmin_pl-1 ) ijm1 = ADM_gmax_pl ! cyclic condition

          dscl_pl(n,k,l) = dscl_pl(n,k,l) &
                         + ( coef_diff_pl(v-1,XDIR,l) * ( vt_pl(ijm1,XDIR) + vt_pl(ij,XDIR) ) &
                           + coef_diff_pl(v-1,YDIR,l) * ( vt_pl(ijm1,YDIR) + vt_pl(ij,YDIR) ) &
                           + coef_diff_pl(v-1,ZDIR,l) * ( vt_pl(ijm1,ZDIR) + vt_pl(ij,ZDIR) ) &
                           ) * 0.5_RP * ( kh_pl(n,k,l) + kh_pl(ij,k,l) )
       enddo

```

```
      enddo
      enddo
   else
      dscl_pl(:,:,:) = 0.0_RP
   endif

   call DEBUG_rapend('OPRT_diffusion')

   return
end subroutine OPRT_diffusion
```

The last part is for calculation for the pole region. Variable `ADM_have_pl` is `.true.` if this process manages pole region in original *NICAM*. For the kernel program, also set as `.true.` in `problem_size.inc`.

2.2.3 Input data and result

Max/min/sum of input/output data of the kernel subroutine are output as a log. Below is an example of `$IAB_SYS=Ubuntu-gnu-ompi` case.

```
### Input ###
+check[check_dscl      ] max=  6.1941315670898286E-08,min= -7.0374144752795210E-08,sum= -2.7407850230958588E-07
+check[check_dscl_pl   ] max=  2.2139244811324830E-08,min= -6.0170678327656930E-11,sum=  1.8274579608905828E-07
+check[scl             ] max=  1.6578626530298903E-11,min= -1.2670212860993856E-11,sum= -2.0289014286353776E-10
+check[scl_pl          ] max=  1.9358849576453664E-11,min= -8.2853331106472899E-12,sum=  1.1445754720677440E-09
+check[kh              ] max=  2.8341305529772246E+12,min=  6.7659597088284981E+10,sum=  6.0439053980501018E+17
+check[kh_pl           ] max=  2.8334094314435532E+12,min=  6.7659597088284981E+10,sum=  4.3486317454839525E+14
### Output ###
+check[dscl            ] max=  6.1941315670898286E-08,min= -7.0374144752795210E-08,sum= -2.7407850230958588E-07
+check[dscl_pl         ] max=  2.2139244811324830E-08,min= -6.0170678327656930E-11,sum=  1.8274579608905828E-07
### Validation : point-by-point diff ###
+check[check_dscl      ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_dscl_pl   ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
*** Finish kernel
```

Check the lines below "`Validation : point-by-point diff`" line, that shows difference between calculated output array and pre-calculated reference array. These should be zero or enough small to be acceptable.

There are sample output log files in `reference/` in each kernel program directory, for reference purpose.

2.2.4 Sample of perfomance result

Here's an example of the performance result part of the log output. Below is an example executed with the machine environment described in subsection 2.1.7. Note that in this program kernel part is iterated one time.

```
*** Computational Time Report
*** ID=001 : MAIN_dyn_diffusion                T=     0.028 N=      1
*** ID=002 : OPRT_diffusion                    T=     0.028 N=      1
```

2.3 `dyn_divdamp`

2.3.1 Description

Kernel `dyn_divdamp` is taken from the original subroutine `OPRT3D_divdamp` in *NICAM*. This subroutine is originally defined, as you can see in that name, in `mod_oprt3d`. Subroutine `OPRT3D_divdamp` calculates the gradient of divergence of the vector $\{v_x, v_y, v_z\}$. If this vector represents the velocity vector, this term is called "divergence damping term".

2.3.2 Discretization and code

Argument lists and local variables definition part of this subroutine is as follows.

```fortran
subroutine OPRT3D_divdamp( &
     ddivdx,    ddivdx_pl,    &
     ddivdy,    ddivdy_pl,    &
     ddivdz,    ddivdz_pl,    &
     rhogvx,    rhogvx_pl,    &
     rhogvy,    rhogvy_pl,    &
     rhogvz,    rhogvz_pl,    &
     rhogw,     rhogw_pl,     &
     coef_intp, coef_intp_pl, &
     coef_diff, coef_diff_pl  )
  implicit none

  real(RP), intent(out) :: ddivdx      (ADM_gall   ,ADM_kall,ADM_lall   ) ! tendency
  real(RP), intent(out) :: ddivdx_pl   (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(out) :: ddivdy      (ADM_gall   ,ADM_kall,ADM_lall   )
  real(RP), intent(out) :: ddivdy_pl   (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(out) :: ddivdz      (ADM_gall   ,ADM_kall,ADM_lall   )
  real(RP), intent(out) :: ddivdz_pl   (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)  :: rhogvx      (ADM_gall   ,ADM_kall,ADM_lall   ) ! rho*vx { gam2 x G^1/2 }
  real(RP), intent(in)  :: rhogvx_pl   (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)  :: rhogvy      (ADM_gall   ,ADM_kall,ADM_lall   ) ! rho*vy { gam2 x G^1/2 }
  real(RP), intent(in)  :: rhogvy_pl   (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)  :: rhogvz      (ADM_gall   ,ADM_kall,ADM_lall   ) ! rho*vz { gam2 x G^1/2 }
  real(RP), intent(in)  :: rhogvz_pl   (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)  :: rhogw       (ADM_gall   ,ADM_kall,ADM_lall   ) ! rho*w  { gam2 x G^1/2 }
  real(RP), intent(in)  :: rhogw_pl    (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)  :: coef_intp   (ADM_gall   ,1:3,ADM_nxyz,TI:TJ,ADM_lall   )
  real(RP), intent(in)  :: coef_intp_pl(ADM_gall_pl,1:3,ADM_nxyz,      ADM_lall_pl)
  real(RP), intent(in)  :: coef_diff   (ADM_gall,1:6        ,ADM_nxyz,ADM_lall   )
  real(RP), intent(in)  :: coef_diff_pl(         1:ADM_vlink,ADM_nxyz,ADM_lall_pl)

  real(RP) :: sclt   (ADM_gall   ,TI:TJ) ! scalar on the hexagon vertex
  real(RP) :: sclt_pl(ADM_gall_pl)
  real(RP) :: sclt_rhogw
  real(RP) :: sclt_rhogw_pl

  real(RP) :: rhogvx_vm   (ADM_gall   )                     ! rho*vx / vertical metrics
  real(RP) :: rhogvx_vm_pl(ADM_gall_pl)
  real(RP) :: rhogvy_vm   (ADM_gall   )                     ! rho*vy / vertical metrics
  real(RP) :: rhogvy_vm_pl(ADM_gall_pl)
  real(RP) :: rhogvz_vm   (ADM_gall   )                     ! rho*vz / vertical metrics
  real(RP) :: rhogvz_vm_pl(ADM_gall_pl)
  real(RP) :: rhogw_vm    (ADM_gall,    ADM_kall,ADM_lall   ) ! rho*w  / vertical metrics
  real(RP) :: rhogw_vm_pl (ADM_gall_pl,ADM_kall,ADM_lall_pl)

  integer  :: gmin, gmax, iall, gall, kall, kmin, kmax, lall, gminm1

  integer  :: ij
  integer  :: ip1j, ijp1, ip1jp1
  integer  :: im1j, ijm1, im1jm1

  integer  :: g, k, l, n, v
  !---------------------------------------------------------------------------
```

Here `ddivdx`, `ddivdy`, `ddivdz` are calculated x, y, z component of the gradient of divergence, respectively. And these with `_pl` are those for the pole region. `rhogvx`, `rhogvy`, `rhogvz`, and `rhogw` are $G^{1/2}\gamma^2 \times \rho v_x$, $G^{1/2}\gamma^2 \times \rho v_y$, $G^{1/2}\gamma^2 \times \rho v_z$, and $G^{1/2}\gamma^2 \times \rho w$, respectively, where $\{v_x, v_y, v_z\}$ are the wind vector of horizontal wind component in 3-D Cartesian coordinates, w is vertical wind, and $G^{1/2}$ and γ are the metrics comes from the terrain-following coordinate described in section 1.6 Other arguments `coef_intp`, `coeff_diff` and those with `_pl` are various coefficients for finite difference calculation, the same with those in `dyn_diffusion`.

Local variable `sclt`, `sclt_pl` are the scalar value on the gravitational cente of triangles, i.e. the vertices of the hexagonal control volume. `rhogvx_vm` etc. are `rhogvx` divided by the vertical metrics.

The first part of this subroutine is as follows.

```fortran
  call DEBUG_rapstart('OPRT3D_divdamp')

  gmin = (ADM_gmin-1)*ADM_gall_1d + ADM_gmin
  gmax = (ADM_gmax-1)*ADM_gall_1d + ADM_gmax
  iall = ADM_gall_1d
  gall = ADM_gall
  kall = ADM_kall
  kmin = ADM_kmin
  kmax = ADM_kmax
```

```
   lall = ADM_lall

   gminm1 = (ADM_gmin-1-1)*ADM_gall_1d + ADM_gmin-1

   !$omp parallel default(none),private(g,k,l), &
   !$omp shared(gall,kmin,kmax,lall,rhogw_vm,rhogvx,rhogvy,rhogvz,rhogw,VMTR_C2WfactGz,VMTR_RGSQRTH,VMTR_RGAMH)
   do l = 1, lall
      !$omp do
      do k = kmin+1, kmax
      do g = 1, gall
         rhogw_vm(g,k,l) = ( VMTR_C2WfactGz(g,k,1,l) * rhogvx(g,k  ,l) &
                           + VMTR_C2WfactGz(g,k,2,l) * rhogvx(g,k-1,l) &
                           + VMTR_C2WfactGz(g,k,3,l) * rhogvy(g,k  ,l) &
                           + VMTR_C2WfactGz(g,k,4,l) * rhogvy(g,k-1,l) &
                           + VMTR_C2WfactGz(g,k,5,l) * rhogvz(g,k  ,l) &
                           + VMTR_C2WfactGz(g,k,6,l) * rhogvz(g,k-1,l) &
                           ) * VMTR_RGAMH(g,k,l)                       & ! horizontal contribution
                         + rhogw(g,k,l) * VMTR_RGSQRTH(g,k,l)            ! vertical   contribution
      enddo
      enddo
      !$omp end do nowait

!OCL XFILL
      !$omp do
      do g = 1, gall
         rhogw_vm(g,kmin  ,l) = 0.0_RP
         rhogw_vm(g,kmax+1,l) = 0.0_RP
      enddo
      !$omp end do
   enddo
   !$omp end parallel

```

This part calculates `rhogw_vm` from 3 components of ρv and ρw (with the metrics). The values are located at the triangluar points in horizonatal direction, and at the half-integer levels in vertical direction. Coefficients prefixed with `VMTR_` are defined and pre-calculated in module `mod_vmtr` in original *NICAM*. When kernelize this subroutine, these definition are moved to `mod_misc`, and read from the input data file.

Second part is pretty long as follows.

```
   !$omp parallel default(none),private(g,k,l,ij,ip1j,ip1jp1,ijp1,im1j,ijm1,im1jm1,sclt_rhogw), &
   !$omp shared(ADM_have_sgp,gminm1,gmin,gmax,gall,kmin,kmax,lall,iall,ddivdx,ddivdy,ddivdz,rhogvx,rhogvy,rhogvz, &
   !$omp        rhogvx_vm,rhogvy_vm,rhogvz_vm,rhogw_vm,sclt,coef_intp,coef_diff,GRD_rdgz,VMTR_RGAM)
   do l = 1, lall
      do k = kmin, kmax
!OCL XFILL
         !$omp do
         do g = 1, gall
            rhogvx_vm(g) = rhogvx(g,k,l) * VMTR_RGAM(g,k,l)
            rhogvy_vm(g) = rhogvy(g,k,l) * VMTR_RGAM(g,k,l)
            rhogvz_vm(g) = rhogvz(g,k,l) * VMTR_RGAM(g,k,l)
         enddo
         !$omp end do

         !$omp do
         do g = gminm1, gmax
            ij     = g
            ip1j   = g + 1
            ip1jp1 = g + iall + 1
            ijp1   = g + iall

            sclt_rhogw = ( ( rhogw_vm(ij,k+1,l) + rhogw_vm(ip1j,k+1,l) + rhogw_vm(ip1jp1,k+1,l) ) &
                         - ( rhogw_vm(ij,k  ,l) + rhogw_vm(ip1j,k  ,l) + rhogw_vm(ip1jp1,k  ,l) ) &
                         ) / 3.0_RP * GRD_rdgz(k)

            sclt(g,TI) = coef_intp(g,1,XDIR,TI,l) * rhogvx_vm(ij    ) &
                       + coef_intp(g,2,XDIR,TI,l) * rhogvx_vm(ip1j  ) &
                       + coef_intp(g,3,XDIR,TI,l) * rhogvx_vm(ip1jp1) &
                       + coef_intp(g,1,YDIR,TI,l) * rhogvy_vm(ij    ) &
                       + coef_intp(g,2,YDIR,TI,l) * rhogvy_vm(ip1j  ) &
                       + coef_intp(g,3,YDIR,TI,l) * rhogvy_vm(ip1jp1) &
                       + coef_intp(g,1,ZDIR,TI,l) * rhogvz_vm(ij    ) &
                       + coef_intp(g,2,ZDIR,TI,l) * rhogvz_vm(ip1j  ) &
                       + coef_intp(g,3,ZDIR,TI,l) * rhogvz_vm(ip1jp1) &
                       + sclt_rhogw
         enddo
         !$omp end do nowait

         !$omp do
```

```
        do g = gminm1, gmax
           ij     = g
           ip1j   = g + 1
           ip1jp1 = g + iall + 1
           ijp1   = g + iall

           sclt_rhogw = ( ( rhogw_vm(ij,k+1,l) + rhogw_vm(ip1jp1,k+1,l) + rhogw_vm(ijp1,k+1,l) ) &
                        - ( rhogw_vm(ij,k  ,l) + rhogw_vm(ip1jp1,k  ,l) + rhogw_vm(ijp1,k  ,l) ) &
                        ) / 3.0_RP * GRD_rdgz(k)

           sclt(g,TJ) = coef_intp(g,1,XDIR,TJ,l) * rhogvx_vm(ij    ) &
                      + coef_intp(g,2,XDIR,TJ,l) * rhogvx_vm(ip1jp1) &
                      + coef_intp(g,3,XDIR,TJ,l) * rhogvx_vm(ijp1  ) &
                      + coef_intp(g,1,YDIR,TJ,l) * rhogvy_vm(ij    ) &
                      + coef_intp(g,2,YDIR,TJ,l) * rhogvy_vm(ip1jp1) &
                      + coef_intp(g,3,YDIR,TJ,l) * rhogvy_vm(ijp1  ) &
                      + coef_intp(g,1,ZDIR,TJ,l) * rhogvz_vm(ij    ) &
                      + coef_intp(g,2,ZDIR,TJ,l) * rhogvz_vm(ip1jp1) &
                      + coef_intp(g,3,ZDIR,TJ,l) * rhogvz_vm(ijp1  ) &
                      + sclt_rhogw
        enddo
        !$omp end do

        if ( ADM_have_sgp(l) ) then ! pentagon
           !$omp master
           sclt(gminm1,TI) = sclt(gminm1+1,TJ)
           !$omp end master
        endif

!OCL XFILL
        !$omp do
        do g = 1, gmin-1
           ddivdx(g,k,l) = 0.0_RP
           ddivdy(g,k,l) = 0.0_RP
           ddivdz(g,k,l) = 0.0_RP
        enddo
        !$omp end do nowait

        !$omp do
        do g = gmin, gmax
           ij     = g
           im1j   = g - 1
           im1jm1 = g - iall - 1
           ijm1   = g - iall

           ddivdx(g,k,l) = ( coef_diff(g,1,XDIR,l) * ( sclt(ij,    TI) + sclt(ij,    TJ) ) &
                           + coef_diff(g,2,XDIR,l) * ( sclt(ij,    TJ) + sclt(im1j,  TI) ) &
                           + coef_diff(g,3,XDIR,l) * ( sclt(im1j,  TI) + sclt(im1jm1,TJ) ) &
                           + coef_diff(g,4,XDIR,l) * ( sclt(im1jm1,TJ) + sclt(im1jm1,TI) ) &
                           + coef_diff(g,5,XDIR,l) * ( sclt(im1jm1,TI) + sclt(ijm1  ,TJ) ) &
                           + coef_diff(g,6,XDIR,l) * ( sclt(ijm1,  TJ) + sclt(ij,    TI) ) )
        enddo
        !$omp end do nowait

        !$omp do
        do g = gmin, gmax
           ij     = g
           im1j   = g - 1
           im1jm1 = g - iall - 1
           ijm1   = g - iall

           ddivdy(g,k,l) = ( coef_diff(g,1,YDIR,l) * ( sclt(ij,    TI) + sclt(ij,    TJ) ) &
                           + coef_diff(g,2,YDIR,l) * ( sclt(ij,    TJ) + sclt(im1j,  TI) ) &
                           + coef_diff(g,3,YDIR,l) * ( sclt(im1j,  TI) + sclt(im1jm1,TJ) ) &
                           + coef_diff(g,4,YDIR,l) * ( sclt(im1jm1,TJ) + sclt(im1jm1,TI) ) &
                           + coef_diff(g,5,YDIR,l) * ( sclt(im1jm1,TI) + sclt(ijm1  ,TJ) ) &
                           + coef_diff(g,6,YDIR,l) * ( sclt(ijm1,  TJ) + sclt(ij,    TI) ) )
        enddo
        !$omp end do nowait

        !$omp do
        do g = gmin, gmax
           ij     = g
           im1j   = g - 1
           im1jm1 = g - iall - 1
           ijm1   = g - iall

           ddivdz(g,k,l) = ( coef_diff(g,1,ZDIR,l) * ( sclt(ij,    TI) + sclt(ij,    TJ) ) &
                           + coef_diff(g,2,ZDIR,l) * ( sclt(ij,    TJ) + sclt(im1j,  TI) ) &
                           + coef_diff(g,3,ZDIR,l) * ( sclt(im1j,  TI) + sclt(im1jm1,TJ) ) &
                           + coef_diff(g,4,ZDIR,l) * ( sclt(im1jm1,TJ) + sclt(im1jm1,TI) ) &
                           + coef_diff(g,5,ZDIR,l) * ( sclt(im1jm1,TI) + sclt(ijm1  ,TJ) ) &
```

```
216                            + coef_diff(g,6,ZDIR,l) * ( sclt(ijm1,  TJ) + sclt(ij,    TI) ) )
217         enddo
218         !$omp end do nowait
219
220 !OCL XFILL
221         !$omp do
222         do g = gmax+1, gall
223            ddivdx(g,k,l) = 0.0_RP
224            ddivdy(g,k,l) = 0.0_RP
225            ddivdz(g,k,l) = 0.0_RP
226         enddo
227         !$omp end do
228      enddo ! loop k
229
230 !OCL XFILL
231      !$omp do
232      do g = 1, gall
233         ddivdx(g,kmin-1,l) = 0.0_RP
234         ddivdy(g,kmin-1,l) = 0.0_RP
235         ddivdz(g,kmin-1,l) = 0.0_RP
236         ddivdx(g,kmax+1,l) = 0.0_RP
237         ddivdy(g,kmax+1,l) = 0.0_RP
238         ddivdz(g,kmax+1,l) = 0.0_RP
239      enddo
240      !$omp end do
241   enddo ! loop l
242   !$omp end parallel
243
```

In the outer most `l`-loop(l.98) and `k`-loop(l.99), `sclt` at the triangular point `TI`(l.120) and `TJ`(l.144) are calculated separately by interpolation. and finally, in the g-loop begins at l.173, desired `ddivdx`(l.179), `ddivdy`(l.195), and `ddivdz`(l.211) are calculated.

The final part is for the pole region, doing almost the same thing as the normal region described above.

```
244   if ( ADM_have_pl ) then
245      n = ADM_gslf_pl
246
247      do l = 1, ADM_lall_pl
248         do k = ADM_kmin+1, ADM_kmax
249         do g = 1, ADM_gall_pl
250            rhogw_vm_pl(g,k,l) = ( VMTR_C2WfactGz_pl(g,k,1,l) * rhogvx_pl(g,k  ,l) &
251                                 + VMTR_C2WfactGz_pl(g,k,2,l) * rhogvx_pl(g,k-1,l) &
252                                 + VMTR_C2WfactGz_pl(g,k,3,l) * rhogvy_pl(g,k  ,l) &
253                                 + VMTR_C2WfactGz_pl(g,k,4,l) * rhogvy_pl(g,k-1,l) &
254                                 + VMTR_C2WfactGz_pl(g,k,5,l) * rhogvz_pl(g,k  ,l) &
255                                 + VMTR_C2WfactGz_pl(g,k,6,l) * rhogvz_pl(g,k-1,l) &
256                                 ) * VMTR_RGAMH_pl(g,k,l)                          & ! horizontal contribution
257                               + rhogw_pl(g,k,l) * VMTR_RGSQRTH_pl(g,k,l)            ! vertical   contribution
258         enddo
259         enddo
260
261         do g = 1, ADM_gall_pl
262            rhogw_vm_pl(g,ADM_kmin  ,l) = 0.0_RP
263            rhogw_vm_pl(g,ADM_kmax+1,l) = 0.0_RP
264         enddo
265      enddo
266
267      do l = 1, ADM_lall_pl
268         do k = ADM_kmin, ADM_kmax
269            do v = 1, ADM_gall_pl
270               rhogvx_vm_pl(v) = rhogvx_pl(v,k,l) * VMTR_RGAM_pl(v,k,l)
271               rhogvy_vm_pl(v) = rhogvy_pl(v,k,l) * VMTR_RGAM_pl(v,k,l)
272               rhogvz_vm_pl(v) = rhogvz_pl(v,k,l) * VMTR_RGAM_pl(v,k,l)
273            enddo
274
275            do v = ADM_gmin_pl, ADM_gmax_pl
276               ij   = v
277               ijp1 = v + 1
278               if( ijp1 == ADM_gmax_pl+1 ) ijp1 = ADM_gmin_pl
279
280               sclt_rhogw_pl = ( ( rhogw_vm_pl(n,k+1,l) + rhogw_vm_pl(ij,k+1,l) + rhogw_vm_pl(ijp1,k+1,l) ) &
281                               - ( rhogw_vm_pl(n,k  ,l) + rhogw_vm_pl(ij,k  ,l) + rhogw_vm_pl(ijp1,k  ,l) ) &
282                               ) / 3.0_RP * GRD_rdgz(k)
283
284               sclt_pl(ij) = coef_intp_pl(v,1,XDIR,l) * rhogvx_vm_pl(n   ) &
285                           + coef_intp_pl(v,2,XDIR,l) * rhogvx_vm_pl(ij  ) &
286                           + coef_intp_pl(v,3,XDIR,l) * rhogvx_vm_pl(ijp1) &
287                           + coef_intp_pl(v,1,YDIR,l) * rhogvy_vm_pl(n   ) &
```

```
                          + coef_intp_pl(v,2,YDIR,l) * rhogvy_vm_pl(ij  ) &
                          + coef_intp_pl(v,3,YDIR,l) * rhogvy_vm_pl(ijp1) &
                          + coef_intp_pl(v,1,ZDIR,l) * rhogvz_vm_pl(n   ) &
                          + coef_intp_pl(v,2,ZDIR,l) * rhogvz_vm_pl(ij  ) &
                          + coef_intp_pl(v,3,ZDIR,l) * rhogvz_vm_pl(ijp1) &
                          + sclt_rhogw_pl
           enddo

           ddivdx_pl(:,k,l) = 0.0_RP
           ddivdy_pl(:,k,l) = 0.0_RP
           ddivdz_pl(:,k,l) = 0.0_RP

           do v = ADM_gmin_pl, ADM_gmax_pl
              ij   = v
              ijm1 = v - 1
              if( ijm1 == ADM_gmin_pl-1 ) ijm1 = ADM_gmax_pl ! cyclic condition

              ddivdx_pl(n,k,l) = ddivdx_pl(n,k,l) + coef_diff_pl(v-1,XDIR,l) * ( sclt_pl(ijm1) + sclt_pl(ij) )
              ddivdy_pl(n,k,l) = ddivdy_pl(n,k,l) + coef_diff_pl(v-1,YDIR,l) * ( sclt_pl(ijm1) + sclt_pl(ij) )
              ddivdz_pl(n,k,l) = ddivdz_pl(n,k,l) + coef_diff_pl(v-1,ZDIR,l) * ( sclt_pl(ijm1) + sclt_pl(ij) )
           enddo
        enddo

        ddivdx_pl(:,ADM_kmin-1,l) = 0.0_RP
        ddivdx_pl(:,ADM_kmax+1,l) = 0.0_RP
        ddivdy_pl(:,ADM_kmin-1,l) = 0.0_RP
        ddivdy_pl(:,ADM_kmax+1,l) = 0.0_RP
        ddivdz_pl(:,ADM_kmin-1,l) = 0.0_RP
        ddivdz_pl(:,ADM_kmax+1,l) = 0.0_RP
     enddo
  else
     ddivdx_pl(:,:,:) = 0.0_RP
     ddivdy_pl(:,:,:) = 0.0_RP
     ddivdz_pl(:,:,:) = 0.0_RP
  endif

  call DEBUG_rapend('OPRT3D_divdamp')

  return
end subroutine OPRT3D_divdamp
```

2.3.3 Input data and result

Max/min/sum of input/output data of the kernel subroutine are output as a log. Below is an example of `$IAB_SYS=Ubuntu-gnu-ompi` case.

```
### Input ###
+check[check_ddivdx    ] max=  3.0131744015374420E-11,min= -5.2988229203876260E-11,sum= -1.6975403236890680E-11
+check[check_ddivdx_pl ] max=  2.6717761683260969E-11,min= -5.3219224048058505E-11,sum=  4.8275913249408648E-11
+check[check_ddivdy    ] max=  2.8051192990274010E-11,min= -2.3223214708700599E-11,sum= -4.5483269668124264E-11
+check[check_ddivdy_pl ] max=  4.4686835079822753E-11,min= -4.1286060234480353E-11,sum=  6.8817604250940581E-11
+check[check_ddivdz    ] max=  3.4380463226247279E-11,min= -1.3010875841849350E-11,sum= -7.2052908719075937E-11
+check[check_ddivdz_pl ] max=  3.2276474319270210E-13,min= -8.5477300588035169E-14,sum=  3.9437219768394675E-12
+check[rhogvx          ] max=  2.2205470252374623E-01,min= -2.0636880346843767E-01,sum= -1.7381945003843336E+02
+check[rhogvx_pl       ] max=  1.4368235944041274E-01,min= -1.9062928892442133E-01,sum= -5.6937904377485706E+00
+check[rhogvy          ] max=  2.4120815977222165E-01,min= -2.5907366818620919E-01,sum=  1.7927398128296358E+02
+check[rhogvy_pl       ] max=  8.8938663283927272E-02,min= -9.6299249362234787E-02,sum= -9.3645167747781997E-03
+check[rhogvz          ] max=  2.0899984486710360E-01,min= -1.9461784957198358E-01,sum=  2.9192900535497301E+02
+check[rhogvz_pl       ] max=  1.2715227826262342E-03,min= -6.6561175383069663E-04,sum=  2.2146129568583188E-02
+check[rhogw           ] max=  1.2675708942695349E-03,min= -4.2678569215437437E-03,sum= -2.6525715444058587E-02
+check[rhogw_pl        ] max=  9.9481836269239257E-04,min= -4.2678569215437437E-03,sum= -8.3801034534239580E-02
### Output ###
+check[ddivdx          ] max=  3.0131744015374420E-11,min= -5.2988229203876260E-11,sum= -1.6975403236890680E-11
+check[ddivdx_pl       ] max=  2.6717761683260969E-11,min= -5.3219224048058505E-11,sum=  4.8275913249408648E-11
+check[ddivdy          ] max=  2.8051192990274010E-11,min= -2.3223214708700599E-11,sum= -4.5483269668124264E-11
+check[ddivdy_pl       ] max=  4.4686835079822753E-11,min= -4.1286060234480353E-11,sum=  6.8817604250940581E-11
+check[ddivdz          ] max=  3.4380463226247279E-11,min= -1.3010875841849350E-11,sum= -7.2052908719075937E-11
+check[ddivdz_pl       ] max=  3.2276474319270210E-13,min= -8.5477300588035169E-14,sum=  3.9437219768394675E-12
### Validation : point-by-point diff ###
+check[check_ddivdx    ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_ddivdx_pl ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_ddivdy    ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_ddivdy_pl ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_ddivdz    ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_ddivdz_pl ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
*** Finish kernel
```

Check the lines below "Validation : point-by-point diff" line, that shows difference between calculated output array and pre-calculated reference array. These should be zero or enough small to be acceptable.

There are sample output log files in reference/ in each kernel program directory, for reference purpose.

2.3.4 Sample of perfomance result

Here's an example of the performance result part of the log output. Below is an example executed with the machine environment described in subsection 2.1.7. Note that in this program kernel part is iterated one time.

```
*** Computational Time Report
*** ID=001 : MAIN_dyn_divdamp                T=     0.028 N=      1
*** ID=002 : OPRT3D_divdamp                  T=     0.028 N=      1
```

2.4 dyn_vi_rhow_solver

2.4.1 Description

Kernel dyn_vi_rhow_solver is taken from the original subroutine vi_rhow_solver in *NICAM*. This subrouine is originally defined in mod_vi. Subroutine vi_rhow_solver is to solve the tridiagonal matrix equations related to the vertical implicit scheme. See section 1.6 for the detail of this calculation.

2.4.2 Discretization and code

Argument lists and local variables definition part of this subroutine is as follows.

```
!-----------------------------------------------------------------------------
!> Tridiagonal matrix solver
subroutine vi_rhow_solver( &
     rhogw,  rhogw_pl,  &
     rhogw0, rhogw0_pl, &
     preg0,  preg0_pl,  &
     rhog0,  rhog0_pl,  &
     Srho,   Srho_pl,   &
     Sw,     Sw_pl,     &
     Spre,   Spre_pl,   &
     dt                 )
  !$ use omp_lib
  implicit none

  real(RP), intent(inout) :: rhogw    (ADM_gall    ,ADM_kall,ADM_lall    ) ! rho*w          ( G^1/2 x gam2 ), n+1
  real(RP), intent(inout) :: rhogw_pl (ADM_gall_pl,ADM_kall,ADM_lall_pl)

  real(RP), intent(in)    :: rhogw0    (ADM_gall    ,ADM_kall,ADM_lall    ) ! rho*w          ( G^1/2 x gam2 )
  real(RP), intent(in)    :: rhogw0_pl(ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)    :: preg0     (ADM_gall    ,ADM_kall,ADM_lall    ) ! pressure prime ( G^1/2 x gam2 )
  real(RP), intent(in)    :: preg0_pl (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)    :: rhog0     (ADM_gall    ,ADM_kall,ADM_lall    ) ! rho            ( G^1/2 x gam2 )
  real(RP), intent(in)    :: rhog0_pl (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)    :: Srho      (ADM_gall    ,ADM_kall,ADM_lall    ) ! source term for rho  at the full level
  real(RP), intent(in)    :: Srho_pl  (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)    :: Sw        (ADM_gall    ,ADM_kall,ADM_lall    ) ! source term for rhow at the half level
  real(RP), intent(in)    :: Sw_pl    (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)    :: Spre      (ADM_gall    ,ADM_kall,ADM_lall    ) ! source term for pres at the full level
  real(RP), intent(in)    :: Spre_pl  (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)    :: dt

  real(RP) :: Sall    (ADM_gall,    ADM_kall)
  real(RP) :: Sall_pl (ADM_gall_pl,ADM_kall)
  real(RP) :: beta    (ADM_gall    )
  real(RP) :: beta_pl (ADM_gall_pl)
  real(RP) :: gamma   (ADM_gall,    ADM_kall)
  real(RP) :: gamma_pl(ADM_gall_pl,ADM_kall)

  integer  :: gall, kmin, kmax, lall
  real(RP) :: grav
  real(RP) :: CVovRt2 ! Cv / R / dt**2
  real(RP) :: alpha
```

```

  integer  :: g, k, l
  integer  :: gstr, gend
  !$ integer  :: n_per_thread
  !$ integer  :: n_thread
  !---------------------------------------------------------------------------

```

Here rhogw is $\rho \times w$ with metric $G^{1/2}\gamma^2$ multiplied at new time step $n+1$. rhogw0, preg0, rhog0, are $\rho \times w$, pressure, ρ with metric multiplied at time step n, respectively. Srho, Sw, Spre are source term for ρ at the full level, source term for $\rho \times w$ at the half level, source term for pressure at the full level, respectively. Other arguments with suffix _pl are for the pole region. dt is a time step for fast-mode.

Among local variables, alpha is the flag for non-hydrostatic/hydrostatic. In this kernel, set to 1 in problem_size.inc.

Main part of this subroutine is as follows.

```
  call DEBUG_rapstart('____vi_rhow_solver')

  gall = ADM_gall
  kmin = ADM_kmin
  kmax = ADM_kmax
  lall = ADM_lall

  grav    = CONST_GRAV
  CVovRt2 = CONST_CVdry / CONST_Rdry / (dt*dt)
  alpha   = real(NON_HYDRO_ALPHA,kind=RP)

  !$omp parallel default(none),private(g,k,l), &
  !$omp private(gstr,gend,n_thread,n_per_thread) &
  !$omp shared(gall,kmin,kmax,lall,rhogw,rhogw0,preg0,rhog0,Srho,Sw,Spre,dt,Sall,beta,gamma,Mu,Mc,Ml, &
  !$omp        GRD_afact,GRD_bfact,GRD_rdgzh,VMTR_GSGAM2H,VMTR_RGAM,VMTR_RGAMH,VMTR_RGSGAM2,VMTR_RGSGAM2H,grav,alpha,CVovRt2)
  gstr = 1
  gend = gall
  !$ n_thread     = omp_get_num_threads()
  !$ n_per_thread = gall / n_thread + int( 0.5_RP + sign(0.5_RP,mod(gall,n_thread)-0.5_RP) )
  !$ gstr         = n_per_thread * omp_get_thread_num() + 1
  !$ gend         = min( gstr+n_per_thread-1, gall )

  do l = 1, lall
     ! calc Sall
     do k = kmin+1, kmax
     do g = gstr, gend
        Sall(g,k) = (   ( rhogw0(g,k,  l)*alpha + dt * Sw   (g,k,  l) ) * VMTR_RGAMH   (g,k,  l)**2            &
                    - ( ( preg0 (g,k,  l)       + dt * Spre(g,k,  l) ) * VMTR_RGSGAM2(g,k,  l)                &
                      - ( preg0 (g,k-1,l)       + dt * Spre(g,k-1,l) ) * VMTR_RGSGAM2(g,k-1,l)                &
                      ) * dt * GRD_rdgzh(k)                                                                   &
                    - ( ( rhog0 (g,k,  l)       + dt * Srho(g,k,  l) ) * VMTR_RGAM(g,k,  l)**2 * GRD_afact(k) &
                      + ( rhog0 (g,k-1,l)       + dt * Srho(g,k-1,l) ) * VMTR_RGAM(g,k-1,l)**2 * GRD_bfact(k) &
                      ) * dt * grav                                                                           &
                    ) * CVovRt2
     enddo
     enddo

     ! boundary conditions
     do g = gstr, gend
        rhogw(g,kmin,  l) = rhogw(g,kmin,  l) * VMTR_RGSGAM2H(g,kmin,  l)
        rhogw(g,kmax+1,l) = rhogw(g,kmax+1,l) * VMTR_RGSGAM2H(g,kmax+1,l)
        Sall (g,kmin+1)   = Sall (g,kmin+1) - Ml(g,kmin+1,l) * rhogw(g,kmin,  l)
        Sall (g,kmax  )   = Sall (g,kmax  ) - Mu(g,kmax,  l) * rhogw(g,kmax+1,l)
     enddo

     !---< solve tri-daigonal matrix >

     ! condition at kmin+1
     k = kmin+1
     do g = gstr, gend
        beta (g)     = Mc(g,k,l)
        rhogw(g,k,l) = Sall(g,k) / beta(g)
     enddo

     ! forward
     do k = kmin+2, kmax
     do g = gstr, gend
        gamma(g,k)   = Mu(g,k-1,l) / beta(g)
        beta (g)     = Mc(g,k,l) - Ml(g,k,l) * gamma(g,k) ! update beta
        rhogw(g,k,l) = ( Sall(g,k) - Ml(g,k,l) * rhogw(g,k-1,l) ) / beta(g)
```

```
110       enddo
111       enddo
112 
113       ! backward
114       do k = kmax-1, kmin+1, -1
115       do g = gstr, gend
116          rhogw(g,k  ,l) = rhogw(g,k  ,l) - gamma(g,k+1) * rhogw(g,k+1,l)
117          rhogw(g,k+1,l) = rhogw(g,k+1,l) * VMTR_GSGAM2H(g,k+1,l) ! return value ( G^1/2 x gam2 )
118       enddo
119       enddo
120 
121       ! boundary treatment
122       do g = gstr, gend
123          rhogw(g,kmin  ,l) = rhogw(g,kmin  ,l) * VMTR_GSGAM2H(g,kmin  ,l)
124          rhogw(g,kmin+1,l) = rhogw(g,kmin+1,l) * VMTR_GSGAM2H(g,kmin+1,l)
125          rhogw(g,kmax+1,l) = rhogw(g,kmax+1,l) * VMTR_GSGAM2H(g,kmax+1,l)
126       enddo
127    enddo
128    !$omp end parallel
```

Inside of the long l-loop(l.72), the first k-loop(l.73) calcurates total source term `Sall`. The section after setting the boundary condition at `kmin` and `kmax` solves tri-diagonal matrix. Its first part(l.97 to l.111) is for forward elimination and the second part(l.114 to l.119) is for backward substitution.

Note that elements of the tridiagonal matrix `Mc`, `Mu` and `Ml` are calculated in advance by other subroutine in the original module, and they are read from input data file in this kernel program.

The last part of this subroutine is for the pole region, doing almost the same process as the normal region.

```
129    if ( ADM_have_pl ) then
130       do l = 1, ADM_lall_pl
131          do k  = ADM_kmin+1, ADM_kmax
132          do g = 1, ADM_gall_pl
133             Sall_pl(g,k) = (   ( rhogw0_pl(g,k,  l)*alpha + dt * Sw_pl  (g,k,  l) ) * VMTR_RGAMH_pl  (g,k,  l)**2            &
134                          - ( ( preg0_pl (g,k,  l)       + dt * Spre_pl(g,k,  l) ) * VMTR_RGSGAM2_pl(g,k,  l)               &
135                            - ( preg0_pl (g,k-1,l)       + dt * Spre_pl(g,k-1,l) ) * VMTR_RGSGAM2_pl(g,k-1,l)               &
136                            ) * dt * GRD_rdgzh(k)                                                                         &
137                          - ( ( rhog0_pl (g,k,  l)       + dt * Srho_pl(g,k,  l) ) * VMTR_RGAM_pl(g,k,  l)**2 * GRD_afact(k) &
138                            + ( rhog0_pl (g,k-1,l)       + dt * Srho_pl(g,k-1,l) ) * VMTR_RGAM_pl(g,k-1,l)**2 * GRD_bfact(k) &
139                            ) * dt * grav                                                                                 &
140                          ) * CVovRt2
141          enddo
142          enddo
143 
144          do g = 1, ADM_gall_pl
145             rhogw_pl(g,ADM_kmin,  l) = rhogw_pl(g,ADM_kmin,  l) * VMTR_RGSGAM2H_pl(g,ADM_kmin,  l)
146             rhogw_pl(g,ADM_kmax+1,l) = rhogw_pl(g,ADM_kmax+1,l) * VMTR_RGSGAM2H_pl(g,ADM_kmax+1,l)
147             Sall_pl (g,ADM_kmin+1)   = Sall_pl (g,ADM_kmin+1) - Ml_pl(g,ADM_kmin+1,l) * rhogw_pl(g,ADM_kmin,  l)
148             Sall_pl (g,ADM_kmax  )   = Sall_pl (g,ADM_kmax  ) - Mu_pl(g,ADM_kmax,  l) * rhogw_pl(g,ADM_kmax+1,l)
149          enddo
150 
151          k = ADM_kmin+1
152          do g = 1, ADM_gall_pl
153             beta_pl (g)     = Mc_pl(g,k,l)
154             rhogw_pl(g,k,l) = Sall_pl(g,k) / beta_pl(g)
155          enddo
156 
157          do k = ADM_kmin+2, ADM_kmax
158          do g = 1, ADM_gall_pl
159             gamma_pl(g,k)   = Mu_pl(g,k-1,l) / beta_pl(g)
160             beta_pl (g)     = Mc_pl(g,k,l) - Ml_pl(g,k,l) * gamma_pl(g,k) ! update beta
161             rhogw_pl(g,k,l) = ( Sall_pl(g,k) - Ml_pl(g,k,l) * rhogw_pl(g,k-1,l) ) / beta_pl(g)
162          enddo
163          enddo
164 
165          ! backward
166          do k = ADM_kmax-1, ADM_kmin+1, -1
167          do g = 1, ADM_gall_pl
168             rhogw_pl(g,k  ,l) = rhogw_pl(g,k  ,l) - gamma_pl(g,k+1) * rhogw_pl(g,k+1,l)
169             rhogw_pl(g,k+1,l) = rhogw_pl(g,k+1,l) * VMTR_GSGAM2H_pl(g,k+1,l) ! return value ( G^1/2 x gam2 )
170          enddo
171          enddo
172 
173          ! boundary treatment
174          do g = 1, ADM_gall_pl
175             rhogw_pl(g,ADM_kmin  ,l) = rhogw_pl(g,ADM_kmin  ,l) * VMTR_GSGAM2H_pl(g,ADM_kmin  ,l)
176             rhogw_pl(g,ADM_kmin+1,l) = rhogw_pl(g,ADM_kmin+1,l) * VMTR_GSGAM2H_pl(g,ADM_kmin+1,l)
177             rhogw_pl(g,ADM_kmax+1,l) = rhogw_pl(g,ADM_kmax+1,l) * VMTR_GSGAM2H_pl(g,ADM_kmax+1,l)
178          enddo
```

```
      enddo
   endif

   call DEBUG_rapend('____vi_rhow_solver')

   return
 end subroutine vi_rhow_solver
```

2.4.3 Input data and result

Max/min/sum of input/output data of the kernel subroutine are output as a log. Below is an example of `$IAB_SYS=Ubuntu-gnu-ompi` case.

```
### Input ###
+check[rhogw_prev       ] max=  1.0733675425174699E-14,min= -2.5212277839542070E-18,sum=  2.1443525022460023E-14
+check[rhogw_prev_pl    ] max=  1.0733675425174699E-14,min= -3.0929371635479732E-15,sum=  7.6415689931329447E-15
+check[check_rhogw      ] max=  6.3321830580406713E-01,min= -5.5759247708875415E-01,sum=  3.9071354372140144E+02
+check[check_rhogw_pl   ] max=  9.9023353298473032E-02,min= -7.4852014128754477E-03,sum=  2.1725659627743767E+00
+check[rhogw0           ] max=  1.2675708942695349E-03,min= -4.2678569215437437E-03,sum= -2.6525715444058587E-02
+check[rhogw0_pl        ] max=  9.9481836269239257E-04,min= -4.2678569215437437E-03,sum= -8.3801034534239580E-02
+check[preg0            ] max=  1.4047194007847271E+01,min= -2.6115802085359952E+01,sum= -1.0687083035942905E+05
+check[preg0_pl         ] max=  1.3572400586711277E+01,min= -2.6115802085359952E+01,sum=  3.4640462790019097E+02
+check[Srho             ] max=  3.3317357550311205E-04,min= -3.9691441465327759E-04,sum= -2.7468694177252501E-01
+check[Srho_pl          ] max=  4.3441154740222974E-05,min= -2.2770470374092324E-05,sum= -3.8685936849254728E-05
+check[Sw               ] max=  2.4651449651180712E-04,min= -3.2516247664737818E-04,sum= -1.6092196259156019E-01
+check[Sw_pl            ] max=  2.8812604579719903E-04,min= -5.5442162236740700E-04,sum= -1.1046915056782920E-02
+check[Spre             ] max=  3.6234075910381584E+01,min= -3.7498614557267203E+01,sum= -2.4763893276893483E+04
+check[Spre_pl          ] max=  4.4863109100507081E+00,min= -2.0915571645010926E+00,sum=  1.0496606720811164E+00
+check[Mc               ] max=  1.7037935104756485E+00,min=  0.0000000000000000E+00,sum=  8.2325595392291399E+05
+check[Mc_pl            ] max=  1.4569727548844720E+00,min=  2.3177911966477871-310,sum=  6.0165678214473996E+02
+check[Ml               ] max=  0.0000000000000000E+00,min= -8.3019108752152282E-01,sum= -3.9659607793632336E+05
+check[Ml_pl            ] max=  3.2282782818256430E+02,min= -6.9953788050738130E-01,sum=  3.4604237063825449E+03
+check[Mu               ] max=  0.0000000000000000E+00,min= -8.6888851023079583E-01,sum= -4.2616616191494197E+05
+check[Mu_pl            ] max=  9.7233873292775215E+01,min= -7.5264063505825385E-01,sum=  8.5169724995286754E+02
### Output ###
+check[rhogw            ] max=  6.3321830580406713E-01,min= -5.5759247708875415E-01,sum=  3.9071354372140144E+02
+check[rhogw_pl         ] max=  9.9023353298473032E-02,min= -7.4852014128754477E-03,sum=  2.1725659627743767E+00
### Validation : point-by-point diff ###
+check[check_rhogw      ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_rhogw_pl   ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
*** Finish kernel
```

Check the lines below "`Validation : point-by-point diff`" line, that shows difference between calculated output array and pre-calculated reference array. These should be zero or enough small to be acceptable.

There are sample output log files in `reference/` in each kernel program directory, for reference purpose.

2.4.4 Sample of perfomance result

Here's an example of the performance result part of the log output. Below is an example executed with the machine environment described in subsection 2.1.7. Note that in this program kernel part is iterated one time.

```
*** Computational Time Report
*** ID=001 : MAIN_dyn_vi_rhow_solver          T=     0.016 N=      1
*** ID=002 : ____vi_rhow_solver               T=     0.016 N=      1
```

2.5 `dyn_vert_adv_limiter`

2.5.1 Description

Kernel `dyn_vert_adv_limiter` is taken from the original subroutine `vertical_limiter_thuburn` in *NICAM*. This subroutine is originally defined in `mod_src_tracer`, that is to contain several subroutines for tracer advection. Subroutine `vertical_limiter_thuburn` is to ensure distribution of tracer quantities' monotonicity in advection scheme, using the flux limitter proposed by Thuburn (1996). This subroutine is for vertical advection only and horizontal advection is treated by other subroutine `horizontal_limiter_thuburn`, which is also kernelized in this pacakage (See section 2.7). See section 4. in Tomita et al. (2010) for details of the tracer scheme in *NICAM*.

2.5.2 Discretization and code

Argument lists and local variables definition part of this subroutine is as follows.

```
subroutine vertical_limiter_thuburn( &
     q_h, q_h_pl, &
     q,   q_pl,   &
     d,   d_pl,   &
     ck,  ck_pl   )
!ESC!    use mod_const, only: &
!ESC!       CONST_HUGE, &
!ESC!       CONST_EPS
!ESC!    use mod_adm, only: &
!ESC!       ADM_have_pl, &
!ESC!       ADM_gall,    &
!ESC!       ADM_gall_pl, &
!ESC!       ADM_lall,    &
!ESC!       ADM_lall_pl, &
!ESC!       ADM_kall,    &
!ESC!       ADM_kmin,    &
!ESC!       ADM_kmax
  implicit none

  real(RP), intent(inout) :: q_h   (ADM_gall   ,ADM_kall,ADM_lall   )
  real(RP), intent(inout) :: q_h_pl(ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)    :: q     (ADM_gall   ,ADM_kall,ADM_lall   )
  real(RP), intent(in)    :: q_pl  (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)    :: d     (ADM_gall   ,ADM_kall,ADM_lall   )
  real(RP), intent(in)    :: d_pl  (ADM_gall_pl,ADM_kall,ADM_lall_pl)
  real(RP), intent(in)    :: ck    (ADM_gall   ,ADM_kall,ADM_lall   ,2)
  real(RP), intent(in)    :: ck_pl (ADM_gall_pl,ADM_kall,ADM_lall_pl,2)

  real(RP) :: Qout_min_k
  real(RP) :: Qout_max_k
  real(RP) :: Qout_min_km1(ADM_gall)
  real(RP) :: Qout_max_km1(ADM_gall)
  real(RP) :: Qout_min_pl(ADM_gall_pl,ADM_kall)
  real(RP) :: Qout_max_pl(ADM_gall_pl,ADM_kall)

  real(RP) :: Qin_minL, Qin_maxL
  real(RP) :: Qin_minU, Qin_maxU
  real(RP) :: qnext_min, qnext_max
  real(RP) :: Cin, Cout
  real(RP) :: CQin_min, CQin_max
  real(RP) :: inflagL, inflagU
  real(RP) :: zerosw

  integer  :: gall, kmin, kmax
  real(RP) :: EPS, BIG

  integer  :: g, k, l
  !---------------------------------------------------------------------------
```

Here `q_h` is q at half level of the vertical layer, which modified by the flux limiter. `q` is q at grid point, `ck` is Courant number, `d` is a correction factor derived from an artificial viscosity for the total density. Note that `ck` has the 4th dimension whose size is 2, which specify lower/upper face, i.e. half integer level.

The first section of the subroutine is as follows.

```
  call DEBUG_rapstart('____vertical_adv_limiter')

  gall = ADM_gall
  kmin = ADM_kmin
  kmax = ADM_kmax

  EPS  = CONST_EPS
  BIG  = CONST_HUGE

  do l = 1, ADM_lall
     !$omp parallel default(none), &
     !$omp private(g,k,zerosw,inflagL,inflagU,Qin_minL,Qin_minU,Qin_maxL,Qin_maxU, &
```

```
    !$omp        qnext_min,qnext_max,Cin,Cout,CQin_min,CQin_max,Qout_min_k,Qout_max_k),  &
    !$omp shared(l,gall,kmin,kmax,q_h,ck,q,d,Qout_min_km1,Qout_max_km1,EPS,BIG)

!OCL XFILL
       !$omp do
       do g = 1, gall
          k = kmin ! peeling

          inflagL = 0.5_RP - sign(0.5_RP,ck(g,k  ,l,1)) ! incoming flux: flag=1
          inflagU = 0.5_RP + sign(0.5_RP,ck(g,k+1,l,1)) ! incoming flux: flag=1

          Qin_minL = min( q(g,k,l), q(g,k-1,l) ) + ( 1.0_RP-inflagL ) * BIG
          Qin_minU = min( q(g,k,l), q(g,k+1,l) ) + ( 1.0_RP-inflagU ) * BIG
          Qin_maxL = max( q(g,k,l), q(g,k-1,l) ) - ( 1.0_RP-inflagL ) * BIG
          Qin_maxU = max( q(g,k,l), q(g,k+1,l) ) - ( 1.0_RP-inflagU ) * BIG

          qnext_min = min( Qin_minL, Qin_minU, q(g,k,l) )
          qnext_max = max( Qin_maxL, Qin_maxU, q(g,k,l) )

          Cin      = (        inflagL ) * ck(g,k,l,1) &
                   + (        inflagU ) * ck(g,k,l,2)
          Cout     = ( 1.0_RP-inflagL ) * ck(g,k,l,1) &
                   + ( 1.0_RP-inflagU ) * ck(g,k,l,2)

          CQin_min = (        inflagL ) * ck(g,k,l,1) * Qin_minL &
                   + (        inflagU ) * ck(g,k,l,2) * Qin_minU
          CQin_max = (        inflagL ) * ck(g,k,l,1) * Qin_maxL &
                   + (        inflagU ) * ck(g,k,l,2) * Qin_maxU

          zerosw = 0.5_RP - sign(0.5_RP,abs(Cout)-EPS) ! if Cout = 0, sw = 1

          Qout_min_k = ( ( q(g,k,l) - qnext_max ) + qnext_max*(Cin+Cout-d(g,k,l)) - CQin_max ) &
                     / ( Cout + zerosw ) * ( 1.0_RP - zerosw )                                 &
                     + q(g,k,l) * zerosw
          Qout_max_k = ( ( q(g,k,l) - qnext_min ) + qnext_min*(Cin+Cout-d(g,k,l)) - CQin_min ) &
                     / ( Cout + zerosw ) * ( 1.0_RP - zerosw )                                 &
                     + q(g,k,l) * zerosw

          Qout_min_km1(g) = Qout_min_k
          Qout_max_km1(g) = Qout_max_k
       enddo
       !$omp end do

       do k = kmin+1, kmax
!OCL XFILL
          !$omp do
          do g = 1, gall
             inflagL = 0.5_RP - sign(0.5_RP,ck(g,k  ,l,1)) ! incoming flux: flag=1
             inflagU = 0.5_RP + sign(0.5_RP,ck(g,k+1,l,1)) ! incoming flux: flag=1

             Qin_minL = min( q(g,k,l), q(g,k-1,l) ) + ( 1.0_RP-inflagL ) * BIG
             Qin_minU = min( q(g,k,l), q(g,k+1,l) ) + ( 1.0_RP-inflagU ) * BIG
             Qin_maxL = max( q(g,k,l), q(g,k-1,l) ) - ( 1.0_RP-inflagL ) * BIG
             Qin_maxU = max( q(g,k,l), q(g,k+1,l) ) - ( 1.0_RP-inflagU ) * BIG

             qnext_min = min( Qin_minL, Qin_minU, q(g,k,l) )
             qnext_max = max( Qin_maxL, Qin_maxU, q(g,k,l) )

             Cin      = (        inflagL ) * ck(g,k,l,1) &
                      + (        inflagU ) * ck(g,k,l,2)
             Cout     = ( 1.0_RP-inflagL ) * ck(g,k,l,1) &
                      + ( 1.0_RP-inflagU ) * ck(g,k,l,2)

             CQin_min = (        inflagL ) * ck(g,k,l,1) * Qin_minL &
                      + (        inflagU ) * ck(g,k,l,2) * Qin_minU
             CQin_max = (        inflagL ) * ck(g,k,l,1) * Qin_maxL &
                      + (        inflagU ) * ck(g,k,l,2) * Qin_maxU

             zerosw = 0.5_RP - sign(0.5_RP,abs(Cout)-EPS) ! if Cout = 0, sw = 1

             Qout_min_k = ( ( q(g,k,l) - qnext_max ) + qnext_max*(Cin+Cout-d(g,k,l)) - CQin_max ) &
                        / ( Cout + zerosw ) * ( 1.0_RP - zerosw )                                 &
                        + q(g,k,l) * zerosw
             Qout_max_k = ( ( q(g,k,l) - qnext_min ) + qnext_min*(Cin+Cout-d(g,k,l)) - CQin_min ) &
                        / ( Cout + zerosw ) * ( 1.0_RP - zerosw )                                 &
                        + q(g,k,l) * zerosw

             q_h(g,k,l) = (        inflagL ) * max( min( q_h(g,k,l), Qout_max_km1(g) ), Qout_min_km1(g) ) &
                        + ( 1.0_RP-inflagL ) * max( min( q_h(g,k,l), Qout_max_k      ), Qout_min_k      )

             Qout_min_km1(g) = Qout_min_k
             Qout_max_km1(g) = Qout_max_k
```

```
        enddo
        !$omp end do
     enddo

   !$omp end parallel
enddo

```

In the long l-loop, there seems to be two blocks, but they are almost the same, except that the first one is only for `kmin`, that is the lowest level, and the second one is the rest of `k` to the top level. `inflagL` and `inflagU` are flag that takes the value 1 if there is an incoming flux to the current layer through the lower/upper face. `Qin_*` are the smaller/larger values of q at lower/upper neighboring layer, that is meaningful only if inflag at lower/upper is 1. `Cin` and `Cout` are the sum of Courant number at both face of the layer, for example, `Cin` is the sum of `ck` at lower face and upper face, if both of `inflagL` and `inflagU` is 1, which means that there are inflow through both lower/upper face. `CQin_*` are the min/max of `Cin` times `Qin_*`, which specify the minimun/maximum of inflow. Then `CQout_*` are calculated. Finaly `q_h` is calculated, which is bounded by `Qout_*`.

The second section of this subroutine is as follows.

```
if ( ADM_have_pl ) then
   do l = 1, ADM_lall_pl

      do k = ADM_kmin, ADM_kmax
        do g = 1, ADM_gall_pl
           inflagL = 0.5_RP - sign(0.5_RP,ck_pl(g,k  ,l,1)) ! incoming flux: flag=1
           inflagU = 0.5_RP + sign(0.5_RP,ck_pl(g,k+1,l,1)) ! incoming flux: flag=1

           Qin_minL = min( q_pl(g,k,l), q_pl(g,k-1,l) ) + ( 1.0_RP-inflagL ) * BIG
           Qin_minU = min( q_pl(g,k,l), q_pl(g,k+1,l) ) + ( 1.0_RP-inflagU ) * BIG
           Qin_maxL = max( q_pl(g,k,l), q_pl(g,k-1,l) ) - ( 1.0_RP-inflagL ) * BIG
           Qin_maxU = max( q_pl(g,k,l), q_pl(g,k+1,l) ) - ( 1.0_RP-inflagU ) * BIG

           qnext_min = min( Qin_minL, Qin_minU, q_pl(g,k,l) )
           qnext_max = max( Qin_maxL, Qin_maxU, q_pl(g,k,l) )

           Cin      = (        inflagL ) * ( ck_pl(g,k,l,1) ) &
                    + (        inflagU ) * ( ck_pl(g,k,l,2) )
           Cout     = ( 1.0_RP-inflagL ) * ( ck_pl(g,k,l,1) ) &
                    + ( 1.0_RP-inflagU ) * ( ck_pl(g,k,l,2) )

           CQin_max = (        inflagL ) * ( ck_pl(g,k,l,1) * Qin_maxL ) &
                    + (        inflagU ) * ( ck_pl(g,k,l,2) * Qin_maxU )
           CQin_min = (        inflagL ) * ( ck_pl(g,k,l,1) * Qin_minL ) &
                    + (        inflagU ) * ( ck_pl(g,k,l,2) * Qin_minU )

           zerosw = 0.5_RP - sign(0.5_RP,abs(Cout)-EPS) ! if Cout = 0, sw = 1

           Qout_min_pl(g,k) = ( ( q_pl(g,k,l) - qnext_max ) + qnext_max*(Cin+Cout-d_pl(g,k,l)) - CQin_max ) &
                            / ( Cout + zerosw ) * ( 1.0_RP - zerosw )                                     &
                            + q_pl(g,k,l) * zerosw
           Qout_max_pl(g,k) = ( ( q_pl(g,k,l) - qnext_min ) + qnext_min*(Cin+Cout-d_pl(g,k,l)) - CQin_min ) &
                            / ( Cout + zerosw ) * ( 1.0_RP - zerosw )                                     &
                            + q_pl(g,k,l) * zerosw
        enddo
        enddo

      do k = ADM_kmin+1, ADM_kmax
      do g = 1, ADM_gall_pl
         inflagL = 0.5_RP - sign(0.5_RP,ck_pl(g,k,l,1)) ! incoming flux: flag=1

         q_h_pl(g,k,l) = (        inflagL ) * max( min( q_h_pl(g,k,l), Qout_max_pl(g,k-1) ), Qout_min_pl(g,k-1) ) &
                       + ( 1.0_RP-inflagL ) * max( min( q_h_pl(g,k,l), Qout_max_pl(g,k  ) ), Qout_min_pl(g,k  ) )
      enddo
      enddo

   enddo
endif

call DEBUG_rapend  ('____vertical_adv_limiter')

return
end subroutine vertical_limiter_thuburn
```

This section is for the pole region, and doing almost the same procedure with the regular region.

2.5.3 Input data and result

Max/min/sum of input/output data of the kernel subroutine are output as a log. Below is an example of `$IAB_SYS=Ubuntu-gnu-ompi` case.

```
### Input ###
+check[q_h_prev        ] max=  7.2663804548391786E+00,min=  0.0000000000000000E+00,sum=  2.4959084727640115E+06
+check[q_h_prev_pl     ] max=  6.4080655438269076E+00,min=  0.0000000000000000E+00,sum=  1.8117616747458535E+03
+check[check_q_h       ] max=  7.3763287914601054E+00,min=  0.0000000000000000E+00,sum=  2.4996023860691669E+06
+check[check_q_h_pl    ] max=  7.0217121722230473E+00,min=  0.0000000000000000E+00,sum=  1.8122948877569047E+03
+check[q               ] max=  7.3763287914601054E+00,min=  0.0000000000000000E+00,sum=  2.4959084727641600E+06
+check[q_pl            ] max=  7.0217121722230473E+00,min=  0.0000000000000000E+00,sum=  1.8117616747458526E+03
+check[d               ] max= -0.0000000000000000E+00,min= -0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[d_pl            ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[ck              ] max=  3.1666977358687308E-02,min= -3.3842023700197510E-02,sum= -3.3888707532669278E+00
+check[ck_pl           ] max=  2.8817336749971920E-02,min= -3.0878557008837175E-02,sum= -2.9483051886829596E-02
### Output ###
+check[q_h             ] max=  7.3763287914601054E+00,min=  0.0000000000000000E+00,sum=  2.4996023860691669E+06
+check[q_h_pl          ] max=  7.0217121722230473E+00,min=  0.0000000000000000E+00,sum=  1.8122948877569047E+03
### Validation : point-by-point diff ###
+check[check_q_h       ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_q_h_pl    ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
*** Finish kernel
```

Check the lines below "`Validation : point-by-point diff`" line, that shows difference between calculated output array and pre-calculated reference array. These should be zero or enough small to be acceptable.

There are sample output log files in `reference/` in each kernel program directory, for reference purpose.

2.5.4 Sample of perfomance result

Here's an example of the performance result part of the log output. Below is an example executed with the machine environment described in subsection 2.1.7. Note that in this program kernel part is iterated one time.

```
*** Computational Time Report
*** ID=001 : MAIN_dyn_vert_adv_limiter        T=      0.032 N=      1
*** ID=002 : ____vertical_adv_limiter         T=      0.032 N=      1
```

2.6 `dyn_horiz_adv_flux`

2.6.1 Description

Kernel `dyn_horiz_adv_flux` is taken from the original subroutine `horizontal_flux` in *NICAM*. This subroutine is originally defined in `mod_src_tracer`. Subroutine `horizontal_flux` is to calculate horizontal advection term, i.e. horizontal mass flux and mass centroid position of an area, which is used for the estimation of mass flux passing through an edge of control cell during one time step. In NICAM, a third order upwind scheme proposed by Miura (2007) is used for the horizontal tracer advection on the icosahedral grid, briefly described in section 1.5. See section 4. in Tomita et al. (2010) for details of tracer scheme in *NICAM*, too.

2.6.2 Discretization and code

Argument lists and local variables definition part of this subroutine is as follows.

```fortran
subroutine horizontal_flux( &
     flx_h,  flx_h_pl,  &
     GRD_xc, GRD_xc_pl, &
     rho,    rho_pl,    &
     rhovx,  rhovx_pl,  &
     rhovy,  rhovy_pl,  &
     rhovz,  rhovz_pl,  &
     dt                 )
  implicit none
```

```fortran

    real(RP), intent(out) :: flx_h    (ADM_gall   ,ADM_kall,ADM_lall   ,6)              ! horizontal mass flux
    real(RP), intent(out) :: flx_h_pl (ADM_gall_pl,ADM_kall,ADM_lall_pl  )
    real(RP), intent(out) :: GRD_xc   (ADM_gall   ,ADM_kall,ADM_lall   ,AI:AJ,XDIR:ZDIR) ! mass centroid position
    real(RP), intent(out) :: GRD_xc_pl(ADM_gall_pl,ADM_kall,ADM_lall_pl,      XDIR:ZDIR)
    real(RP), intent(in)  :: rho      (ADM_gall   ,ADM_kall,ADM_lall   )                ! rho at cell center
    real(RP), intent(in)  :: rho_pl   (ADM_gall_pl,ADM_kall,ADM_lall_pl)
    real(RP), intent(in)  :: rhovx    (ADM_gall   ,ADM_kall,ADM_lall   )
    real(RP), intent(in)  :: rhovx_pl (ADM_gall_pl,ADM_kall,ADM_lall_pl)
    real(RP), intent(in)  :: rhovy    (ADM_gall   ,ADM_kall,ADM_lall   )
    real(RP), intent(in)  :: rhovy_pl (ADM_gall_pl,ADM_kall,ADM_lall_pl)
    real(RP), intent(in)  :: rhovz    (ADM_gall   ,ADM_kall,ADM_lall   )
    real(RP), intent(in)  :: rhovz_pl (ADM_gall_pl,ADM_kall,ADM_lall_pl)
    real(RP), intent(in)  :: dt

    real(RP) :: rhot_TI  (ADM_gall   ) ! rho at cell vertex
    real(RP) :: rhot_TJ  (ADM_gall   ) ! rho at cell vertex
    real(RP) :: rhot_pl  (ADM_gall_pl)
    real(RP) :: rhovxt_TI(ADM_gall   )
    real(RP) :: rhovxt_TJ(ADM_gall   )
    real(RP) :: rhovxt_pl(ADM_gall_pl)
    real(RP) :: rhovyt_TI(ADM_gall   )
    real(RP) :: rhovyt_TJ(ADM_gall   )
    real(RP) :: rhovyt_pl(ADM_gall_pl)
    real(RP) :: rhovzt_TI(ADM_gall   )
    real(RP) :: rhovzt_TJ(ADM_gall   )
    real(RP) :: rhovzt_pl(ADM_gall_pl)

    real(RP) :: rhovxt2
    real(RP) :: rhovyt2
    real(RP) :: rhovzt2
    real(RP) :: flux
    real(RP) :: rrhoa2

    integer  :: gmin, gmax, kall, iall
    real(RP) :: EPS

    integer  :: ij
    integer  :: ip1j, ijp1, ip1jp1
    integer  :: im1j, ijm1

    integer  :: i, j, k, l, n, v
    !---------------------------------------------------------------------------
```

Output variable `flx_h` is horizontal mass flux and `GRD_xc` is the spatial position C_i in Figure 1.5(b). Note that `flx_h` has 4th dimension whose size is 6, specifying 6 edges of hexagon control volume. Similarly, `GRD_xc` has 4th dimension ranged `(AI:AJ)` and 5th dimension ranged `(XDIR:ZDIR)`, the former specifies three arc points and the latter specifies 3 coodinates of the position. Input variable `rho`, `rhovx`, `rhovy` and `rhovz` are ρ, ρv_x, ρv_y, and ρv_z at cell center with metrics multiplied, respectively. Other arguments with suffix `_pl` are for the pole region.

Among local variables, `rhot_TI`, `rhot_TJ` are interpolated ρ at the gravitational center of downward triangle and upward triangle, respectively. Other variables `rho*` with suffix `_TI`, `_TJ` are the same.

The first half of main part is as follows.

```fortran
    call DEBUG_rapstart('____horizontal_adv_flux')

    gmin = ADM_gmin
    gmax = ADM_gmax
    kall = ADM_kall
    iall = ADM_gall_1d

    EPS  = CONST_EPS

    do l = 1, ADM_lall
       !$omp parallel default(none), &
       !$omp private(i,j,k,ij,ip1j,ip1jp1,ijp1,im1j,ijm1,                                &
       !$omp         rrhoa2,rhovxt2,rhovyt2,rhovzt2,flux),                               &
       !$omp shared(l,ADM_have_sgp,gmin,gmax,kall,iall,rho,rhovx,rhovy,rhovz,flx_h,dt,         &
       !$omp        rhot_TI,rhovxt_TI,rhovyt_TI,rhovzt_TI,rhot_TJ,rhovxt_TJ,rhovyt_TJ,rhovzt_TJ, &
       !$omp        GRD_xc,GRD_xr,GMTR_p,GMTR_t,GMTR_a,EPS)
       do k = 1, kall

          ! (i,j),(i+1,j)
          !$omp do
          do j = gmin-1, gmax
          do i = gmin-1, gmax
```

```
75           ij     = (j-1)*iall + i
76           ip1j   = ij + 1
77
78           rhot_TI  (ij) = rho  (ij  ,k,l) * GMTR_t(ij,K0,l,TI,W1) &
79                         + rho  (ip1j,k,l) * GMTR_t(ij,K0,l,TI,W2)
80           rhovxt_TI(ij) = rhovx(ij  ,k,l) * GMTR_t(ij,K0,l,TI,W1) &
81                         + rhovx(ip1j,k,l) * GMTR_t(ij,K0,l,TI,W2)
82           rhovyt_TI(ij) = rhovy(ij  ,k,l) * GMTR_t(ij,K0,l,TI,W1) &
83                         + rhovy(ip1j,k,l) * GMTR_t(ij,K0,l,TI,W2)
84           rhovzt_TI(ij) = rhovz(ij  ,k,l) * GMTR_t(ij,K0,l,TI,W1) &
85                         + rhovz(ip1j,k,l) * GMTR_t(ij,K0,l,TI,W2)
86
87           rhot_TJ  (ij) = rho  (ij  ,k,l) * GMTR_t(ij,K0,l,TJ,W1)
88           rhovxt_TJ(ij) = rhovx(ij  ,k,l) * GMTR_t(ij,K0,l,TJ,W1)
89           rhovyt_TJ(ij) = rhovy(ij  ,k,l) * GMTR_t(ij,K0,l,TJ,W1)
90           rhovzt_TJ(ij) = rhovz(ij  ,k,l) * GMTR_t(ij,K0,l,TJ,W1)
91        enddo
92        enddo
93        !$omp end do
94
95        ! (i,j+1),(i+1,j+1)
96        !$omp do
97        do j = gmin-1, gmax
98        do i = gmin-1, gmax
99           ij     = (j-1)*iall + i
100          ijp1   = ij + iall
101          ip1jp1 = ij + iall + 1
102
103          rhot_TI  (ij) = rhot_TI  (ij) + rho  (ip1jp1,k,l) * GMTR_t(ij,K0,l,TI,W3)
104          rhovxt_TI(ij) = rhovxt_TI(ij) + rhovx(ip1jp1,k,l) * GMTR_t(ij,K0,l,TI,W3)
105          rhovyt_TI(ij) = rhovyt_TI(ij) + rhovy(ip1jp1,k,l) * GMTR_t(ij,K0,l,TI,W3)
106          rhovzt_TI(ij) = rhovzt_TI(ij) + rhovz(ip1jp1,k,l) * GMTR_t(ij,K0,l,TI,W3)
107
108          rhot_TJ  (ij) = rhot_TJ  (ij) + rho  (ip1jp1,k,l) * GMTR_t(ij,K0,l,TJ,W2) &
109                                        + rho  (ijp1  ,k,l) * GMTR_t(ij,K0,l,TJ,W3)
110          rhovxt_TJ(ij) = rhovxt_TJ(ij) + rhovx(ip1jp1,k,l) * GMTR_t(ij,K0,l,TJ,W2) &
111                                        + rhovx(ijp1  ,k,l) * GMTR_t(ij,K0,l,TJ,W3)
112          rhovyt_TJ(ij) = rhovyt_TJ(ij) + rhovy(ip1jp1,k,l) * GMTR_t(ij,K0,l,TJ,W2) &
113                                        + rhovy(ijp1  ,k,l) * GMTR_t(ij,K0,l,TJ,W3)
114          rhovzt_TJ(ij) = rhovzt_TJ(ij) + rhovz(ip1jp1,k,l) * GMTR_t(ij,K0,l,TJ,W2) &
115                                        + rhovz(ijp1  ,k,l) * GMTR_t(ij,K0,l,TJ,W3)
116       enddo
117       enddo
118       !$omp end do
119
120       if ( ADM_have_sgp(l) ) then
121          !$omp master
122          j = gmin-1
123          i = gmin-1
124
125          ij   = (j-1)*iall + i
126          ip1j = ij + 1
127
128          rhot_TI  (ij) = rhot_TJ  (ip1j)
129          rhovxt_TI(ij) = rhovxt_TJ(ip1j)
130          rhovyt_TI(ij) = rhovyt_TJ(ip1j)
131          rhovzt_TI(ij) = rhovzt_TJ(ip1j)
132          !$omp end master
133       endif
134
```

In long l-loop and k-loop, first part calculate ρ, ρv_x, ρv_y, ρv_z at two center points of triangle at `TI` and `TJ`. Note that two triangular points are surrounded by 4 grid points (i, j), $(i+1, j)$, $(i+1, j+1)$ and $(i, j+1)$. The first i, j-double loop(l.73-) calculates contribution from the former two grid points, and the second i, j-double loop(l.97-) does from the latter two grid points. `IF` clause from l.120 is treatment for the singular point. `GMTR_t` is the metrics for triangle linear interpolation from three trianglar vertices to gravitational center point of triangle. In original *NICAM*, this array is defined as `GMTR_T_var` in module `mod_gmtr`. In this kernel program, this is read from input data file. Note that `TI`, `TJ`, `W1`, `W2` and `W3` are not loop index but constant defined in `problem_size.inc`, those are originally defined in `mod_gmtr` in *NICAM*.

The second half of main part is as follows.

```
135       !--- calculate flux and mass centroid position
136
137 !OCL XFILL
138       !$omp do
139       do j = 1, iall
```

```
      do i = 1, iall
         if (      i < gmin .OR. i > gmax &
              .OR. j < gmin .OR. j > gmax ) then
            ij = (j-1)*iall + i

            flx_h(ij,k,l,1) = 0.0_RP
            flx_h(ij,k,l,2) = 0.0_RP
            flx_h(ij,k,l,3) = 0.0_RP
            flx_h(ij,k,l,4) = 0.0_RP
            flx_h(ij,k,l,5) = 0.0_RP
            flx_h(ij,k,l,6) = 0.0_RP

            GRD_xc(ij,k,l,AI ,XDIR) = 0.0_RP
            GRD_xc(ij,k,l,AI ,YDIR) = 0.0_RP
            GRD_xc(ij,k,l,AI ,ZDIR) = 0.0_RP
            GRD_xc(ij,k,l,AIJ,XDIR) = 0.0_RP
            GRD_xc(ij,k,l,AIJ,YDIR) = 0.0_RP
            GRD_xc(ij,k,l,AIJ,ZDIR) = 0.0_RP
            GRD_xc(ij,k,l,AJ ,XDIR) = 0.0_RP
            GRD_xc(ij,k,l,AJ ,YDIR) = 0.0_RP
            GRD_xc(ij,k,l,AJ ,ZDIR) = 0.0_RP
         endif
      enddo
      enddo
      !$omp end do

      !$omp do
      do j = gmin  , gmax
      do i = gmin-1, gmax
         ij     = (j-1)*iall + i
         ip1j   = ij + 1
         ijm1   = ij - iall

         rrhoa2  = 1.0_RP / max( rhot_TJ(ijm1) + rhot_TI(ij), EPS ) ! doubled
         rhovxt2 = rhovxt_TJ(ijm1) + rhovxt_TI(ij)
         rhovyt2 = rhovyt_TJ(ijm1) + rhovyt_TI(ij)
         rhovzt2 = rhovzt_TJ(ijm1) + rhovzt_TI(ij)

         flux = 0.5_RP * ( rhovxt2 * GMTR_a(ij,k0,l,AI ,HNX) &
                         + rhovyt2 * GMTR_a(ij,k0,l,AI ,HNY) &
                         + rhovzt2 * GMTR_a(ij,k0,l,AI ,HNZ) )

         flx_h(ij  ,k,l,1) =  flux * GMTR_p(ij  ,k0,l,P_RAREA) * dt
         flx_h(ip1j,k,l,4) = -flux * GMTR_p(ip1j,k0,l,P_RAREA) * dt

         GRD_xc(ij,k,l,AI,XDIR) = GRD_xr(ij,K0,l,AI,XDIR) - rhovxt2 * rrhoa2 * dt * 0.5_RP
         GRD_xc(ij,k,l,AI,YDIR) = GRD_xr(ij,K0,l,AI,YDIR) - rhovyt2 * rrhoa2 * dt * 0.5_RP
         GRD_xc(ij,k,l,AI,ZDIR) = GRD_xr(ij,K0,l,AI,ZDIR) - rhovzt2 * rrhoa2 * dt * 0.5_RP
      enddo
      enddo
      !$omp end do

      !$omp do
      do j = gmin-1, gmax
      do i = gmin-1, gmax
         ij     = (j-1)*iall + i
         ip1jp1 = ij + iall + 1

         rrhoa2  = 1.0_RP / max( rhot_TI(ij) + rhot_TJ(ij), EPS ) ! doubled
         rhovxt2 = rhovxt_TI(ij) + rhovxt_TJ(ij)
         rhovyt2 = rhovyt_TI(ij) + rhovyt_TJ(ij)
         rhovzt2 = rhovzt_TI(ij) + rhovzt_TJ(ij)

         flux = 0.5_RP * ( rhovxt2 * GMTR_a(ij,k0,l,AIJ,HNX) &
                         + rhovyt2 * GMTR_a(ij,k0,l,AIJ,HNY) &
                         + rhovzt2 * GMTR_a(ij,k0,l,AIJ,HNZ) )

         flx_h(ij    ,k,l,2) =  flux * GMTR_p(ij    ,k0,l,P_RAREA) * dt
         flx_h(ip1jp1,k,l,5) = -flux * GMTR_p(ip1jp1,k0,l,P_RAREA) * dt

         GRD_xc(ij,k,l,AIJ,XDIR) = GRD_xr(ij,K0,l,AIJ,XDIR) - rhovxt2 * rrhoa2 * dt * 0.5_RP
         GRD_xc(ij,k,l,AIJ,YDIR) = GRD_xr(ij,K0,l,AIJ,YDIR) - rhovyt2 * rrhoa2 * dt * 0.5_RP
         GRD_xc(ij,k,l,AIJ,ZDIR) = GRD_xr(ij,K0,l,AIJ,ZDIR) - rhovzt2 * rrhoa2 * dt * 0.5_RP
      enddo
      enddo
      !$omp end do

      !$omp do
      do j = gmin-1, gmax
      do i = gmin  , gmax
         ij     = (j-1)*iall + i
         ijp1   = ij + iall
```

```
            im1j   = ij - 1

            rrhoa2  = 1.0_RP / max( rhot_TJ(ij) + rhot_TI(im1j), EPS ) ! doubled
            rhovxt2 = rhovxt_TJ(ij) + rhovxt_TI(im1j)
            rhovyt2 = rhovyt_TJ(ij) + rhovyt_TI(im1j)
            rhovzt2 = rhovzt_TJ(ij) + rhovzt_TI(im1j)

            flux = 0.5_RP * ( rhovxt2 * GMTR_a(ij,k0,l,AJ ,HNX) &
                            + rhovyt2 * GMTR_a(ij,k0,l,AJ ,HNY) &
                            + rhovzt2 * GMTR_a(ij,k0,l,AJ ,HNZ) )

            flx_h(ij  ,k,l,3) =  flux * GMTR_p(ij  ,k0,l,P_RAREA) * dt
            flx_h(ijp1,k,l,6) = -flux * GMTR_p(ijp1,k0,l,P_RAREA) * dt

            GRD_xc(ij,k,l,AJ,XDIR) = GRD_xr(ij,K0,l,AJ,XDIR) - rhovxt2 * rrhoa2 * dt * 0.5_RP
            GRD_xc(ij,k,l,AJ,YDIR) = GRD_xr(ij,K0,l,AJ,YDIR) - rhovyt2 * rrhoa2 * dt * 0.5_RP
            GRD_xc(ij,k,l,AJ,ZDIR) = GRD_xr(ij,K0,l,AJ,ZDIR) - rhovzt2 * rrhoa2 * dt * 0.5_RP
         enddo
         enddo
         !$omp end do

         if ( ADM_have_sgp(l) ) then
            !$omp master
            j = gmin
            i = gmin

            ij = (j-1)*iall + i

            flx_h(ij,k,l,6) = 0.0_RP
            !$omp end master
         endif

      enddo
      !$omp end parallel
   enddo

```

There are 4 i,j-double loops in the long k and l loop continued from previous section. After setting halo region as 0.0, each 3 loops calculates`flx_h` and `GRD_xc` at each 3 arc points specified by `AI`, `AIJ`, `AJ`. Here `GMTR_a` is the metrics for the arc points (the normal vector on the arc point). In original *NICAM*, the array is defined as `GMTR_A_var` in module `mod_gmtr`. In this kernel program, this is read from input data file. Similarly, `GMTR_p` is the metrics for grid points (the reciprocal number of the area of the control cell).

The last part is for the pole region, doing almost same calculation with the normal region.

```
   if ( ADM_have_pl ) then
      n = ADM_gslf_pl

      do l = 1, ADM_lall_pl
      do k = 1, ADM_kall

         do v = ADM_gmin_pl, ADM_gmax_pl
            ij   = v
            ijp1 = v + 1
            if( ijp1 == ADM_gmax_pl + 1 ) ijp1 = ADM_gmin_pl

            rhot_pl  (v) = rho_pl  (n   ,k,l) * GMTR_t_pl(ij,K0,l,W1) &
                         + rho_pl  (ij  ,k,l) * GMTR_t_pl(ij,K0,l,W2) &
                         + rho_pl  (ijp1,k,l) * GMTR_t_pl(ij,K0,l,W3)
            rhovxt_pl(v) = rhovx_pl(n   ,k,l) * GMTR_t_pl(ij,K0,l,W1) &
                         + rhovx_pl(ij  ,k,l) * GMTR_t_pl(ij,K0,l,W2) &
                         + rhovx_pl(ijp1,k,l) * GMTR_t_pl(ij,K0,l,W3)
            rhovyt_pl(v) = rhovy_pl(n   ,k,l) * GMTR_t_pl(ij,K0,l,W1) &
                         + rhovy_pl(ij  ,k,l) * GMTR_t_pl(ij,K0,l,W2) &
                         + rhovy_pl(ijp1,k,l) * GMTR_t_pl(ij,K0,l,W3)
            rhovzt_pl(v) = rhovz_pl(n   ,k,l) * GMTR_t_pl(ij,K0,l,W1) &
                         + rhovz_pl(ij  ,k,l) * GMTR_t_pl(ij,K0,l,W2) &
                         + rhovz_pl(ijp1,k,l) * GMTR_t_pl(ij,K0,l,W3)
         enddo

         do v = ADM_gmin_pl, ADM_gmax_pl
            ij   = v
            ijm1 = v - 1
            if( ijm1 == ADM_gmin_pl - 1 ) ijm1 = ADM_gmax_pl

            rrhoa2  = 1.0_RP / max( rhot_pl(ijm1) + rhot_pl(ij), EPS ) ! doubled
            rhovxt2 = rhovxt_pl(ijm1) + rhovxt_pl(ij)
            rhovyt2 = rhovyt_pl(ijm1) + rhovyt_pl(ij)
```

```
            rhovzt2 = rhovzt_pl(ijm1) + rhovzt_pl(ij)

            flux = 0.5_RP * ( rhovxt2 * GMTR_a_pl(ij,K0,l,HNX) &
                            + rhovyt2 * GMTR_a_pl(ij,K0,l,HNY) &
                            + rhovzt2 * GMTR_a_pl(ij,K0,l,HNZ) )

            flx_h_pl(v,k,l) = flux * GMTR_p_pl(n,K0,l,P_RAREA) * dt

            GRD_xc_pl(v,k,l,XDIR) = GRD_xr_pl(v,K0,l,XDIR) - rhovxt2 * rrhoa2 * dt * 0.5_RP
            GRD_xc_pl(v,k,l,YDIR) = GRD_xr_pl(v,K0,l,YDIR) - rhovyt2 * rrhoa2 * dt * 0.5_RP
            GRD_xc_pl(v,k,l,ZDIR) = GRD_xr_pl(v,K0,l,ZDIR) - rhovzt2 * rrhoa2 * dt * 0.5_RP
         enddo

      enddo
      enddo
   endif

   call DEBUG_rapend  ('____horizontal_adv_flux')

   return
 end subroutine horizontal_flux
```

2.6.3 Input data and result

Max/min/sum of input/output data of the kernel subroutine are output as a log. Below is an example of `$IAB_SYS=Ubuntu-gnu-ompi` case.

```
### Input ###
+check[flx_h          ] max=  1.5610710092943020E-01,min= -1.5670677438306818E-01,sum=  2.5627807148368471E+00
+check[flx_h_pl       ] max=  1.1002449397299904E-01,min= -1.0379423223736732E-01,sum=  2.3428578735122479E-02
+check[GRD_xc         ] max=  6.3711740974481208E+06,min= -2.8421843880731151E+06,sum=  2.0008127660775812E+13
+check[GRD_xc_pl      ] max=  6.3711707368004546E+06,min= -6.3711702446065033E+06,sum= -5.2402711763852555E+05
+check[rhog_mean      ] max=  1.3664531579209602E+00,min=  3.0688241919459083E-02,sum=  3.0079615977439255E+05
+check[rhog_mean_pl   ] max=  1.3664531579209602E+00,min=  3.0688131204860365E-02,sum=  2.1686973266477614E+02
+check[rhogvx         ] max=  2.4807620495454326E+01,min= -2.6743430307618826E+01,sum= -7.2284486916298035E+05
+check[rhogvx_pl      ] max=  2.3291561831945440E+01,min= -1.7023283571413913E+01,sum=  7.6579748418940346E+02
+check[rhogvy         ] max=  3.6016622784129858E+01,min= -3.3418704823922688E+01,sum=  8.9473900044437428E+05
+check[rhogvy_pl      ] max=  8.7323000593655760E+00,min= -9.4486634372500102E+00,sum=  1.2151517834227268E+03
+check[rhogvz         ] max=  2.3202617358688446E+01,min= -2.4977800359078525E+01,sum=  1.1280540293791931E+05
+check[rhogvz_pl      ] max=  1.6015604765102806E-01,min= -1.3081853271482025E-01,sum=  1.6285945760677518E+00
+check[GRD_xr         ] max=  4.0592444288400000E+13,min= -9.9998999999999996E+30,sum= -1.5719842799999647E+34
+check[GRD_xr_pl      ] max=  6.3711570243787766E+06,min= -9.9998999999999996E+30,sum= -5.9999399999999993E+31
+check[GMTR_p         ] max=  3.1937623126446486E+09,min= -1.0000000000000000E+00,sum=  5.2568771941580258E+13
+check[GMTR_p_pl      ] max=  2.6945919723895960E+09,min= -1.0000000000000000E+00,sum=  3.0761871439307461E+10
+check[GMTR_t         ] max=  1.5988904134747729E+09,min=  0.0000000000000000E+00,sum=  5.1787428410119117E+13
+check[GMTR_t_pl      ] max=  1.9722549493151660E+09,min=  0.0000000000000000E+00,sum=  1.9722549503025471E+10
+check[GMTR_a         ] max=  6.3362880879999531E+04,min= -6.0857794015098916E+04,sum=  7.9258961023823655E+08
+check[GMTR_a_pl      ] max=  5.7873883883004550E+04,min= -5.7872753043726123E+04,sum= -4.3064210331067443E-07
### Output ###
+check[flx_h          ] max=  1.5610710092943020E-01,min= -1.5670677438306818E-01,sum=  2.5627807148368471E+00
+check[flx_h_pl       ] max=  1.1002449397299904E-01,min= -1.0379423223736732E-01,sum=  2.3428578735122479E-02
+check[GRD_xc         ] max=  6.3711740974481208E+06,min= -2.8421843880731151E+06,sum=  2.0008127660775812E+13
+check[GRD_xc_pl      ] max=  6.3711707368004546E+06,min= -6.3711702446065033E+06,sum= -5.2402711763852555E+05
### Validation : point-by-point diff ###
+check[check_flx_h    ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_flx_h_pl ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_GRD_xc   ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_GRD_xc_pl ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
*** Finish kernel
```

Check the lines below “`Validation : point-by-point diff`” line, that shows difference between calculated output array and pre-calculated reference array. These should be zero or enough small to be acceptable.

There are sample output log files in `reference/` in each kernel program directory, for reference purpose.

2.6.4 Sample of perfomance result

Here’s an example of the performance result part of the log output. Below is an example executed with the machine environment described in subsection 2.1.7. Note that in this program kernel part is iterated one time.

```
*** Computational Time Report
*** ID=001 : MAIN_dyn_horiz_adv_flux         T=     0.040 N=      1
*** ID=002 : ____horizontal_adv_flux         T=     0.040 N=      1
```

2.7 dyn_horiz_adv_limiter

2.7.1 Description

Kernel dyn_horiz_adv_limiter is taken from the original subroutine horizontal_limiter_thuburn in *NICAM*. This subroutine is originally defined in mod_src_tracer, that is to contain several subroutines for tracer advection. Subroutine horizontal_limiter_thuburn is to ensure distribution of tracer quantities' monotonicity in advection scheme, using the flux limitter proposed by Thuburn (1996). This subroutine is for horizontal advection only and vertical advection is treated by other subroutine vertical_limiter_thuburn, which is also kernelized in this package (See section 2.5). In NICAM, a third order upwind scheme proposed by Miura (2007) is used for the horizontal tracer advection on the icosahedral grid. See section 4. in Tomita et al. (2010) for details of the tracer scheme in *NICAM*, too.

2.7.2 Discretization and code

Argument lists and local variables definition part of this subroutine is as follows.

```
subroutine horizontal_limiter_thuburn( &
     q_a,    q_a_pl,  &
     q,      q_pl,    &
     d,      d_pl,    &
     ch,     ch_pl,   &
     cmask,  cmask_pl, &
     Qout_prev, Qout_prev_pl, & ! KERNEL
     Qout_post, Qout_post_pl  ) ! KERNEL
!ESC!    use mod_const, only: &
!ESC!       CONST_HUGE, &
!ESC!       CONST_EPS
!ESC!    use mod_adm, only: &
!ESC!       ADM_have_pl,    &
!ESC!       ADM_have_sgp,   &
!ESC!       ADM_lall,       &
!ESC!       ADM_lall_pl,    &
!ESC!       ADM_gall,       &
!ESC!       ADM_gall_pl,    &
!ESC!       ADM_kall,       &
!ESC!       ADM_gall_1d,    &
!ESC!       ADM_gmin,       &
!ESC!       ADM_gmax,       &
!ESC!       ADM_gslf_pl,    &
!ESC!       ADM_gmin_pl,    &
!ESC!       ADM_gmax_pl
!ESC!    use mod_comm, only: &
!ESC!       COMM_data_transfer
!ESC!    implicit none

  real(RP), intent(inout) :: q_a     (ADM_gall   ,ADM_kall,ADM_lall   ,6)
  real(RP), intent(inout) :: q_a_pl  (ADM_gall_pl,ADM_kall,ADM_lall_pl  )
  real(RP), intent(in)    :: q       (ADM_gall   ,ADM_kall,ADM_lall     )
  real(RP), intent(in)    :: q_pl    (ADM_gall_pl,ADM_kall,ADM_lall_pl  )
  real(RP), intent(in)    :: d       (ADM_gall   ,ADM_kall,ADM_lall     )
  real(RP), intent(in)    :: d_pl    (ADM_gall_pl,ADM_kall,ADM_lall_pl  )
  real(RP), intent(in)    :: ch      (ADM_gall   ,ADM_kall,ADM_lall   ,6)
  real(RP), intent(in)    :: ch_pl   (ADM_gall_pl,ADM_kall,ADM_lall_pl  )
  real(RP), intent(in)    :: cmask   (ADM_gall   ,ADM_kall,ADM_lall   ,6)
  real(RP), intent(in)    :: cmask_pl(ADM_gall_pl,ADM_kall,ADM_lall_pl  )
  real(RP), intent(out)   :: Qout_prev   (ADM_gall   ,ADM_kall,ADM_lall   ,2 ) ! before communication (for check)
  real(RP), intent(out)   :: Qout_prev_pl(ADM_gall_pl,ADM_kall,ADM_lall_pl,2 ) !
  real(RP), intent(in)    :: Qout_post   (ADM_gall   ,ADM_kall,ADM_lall   ,2 ) ! after communication (additional input)
  real(RP), intent(in)    :: Qout_post_pl(ADM_gall_pl,ADM_kall,ADM_lall_pl,2 ) !

  real(RP) :: q_min_AI, q_min_AIJ, q_min_AJ, q_min_pl
  real(RP) :: q_max_AI, q_max_AIJ, q_max_AJ, q_max_pl

  real(RP) :: qnext_min   , qnext_min_pl
  real(RP) :: qnext_max   , qnext_max_pl
```

```
  real(RP) :: Cin_sum     , Cin_sum_pl
  real(RP) :: Cout_sum    , Cout_sum_pl
  real(RP) :: CQin_max_sum, CQin_max_sum_pl
  real(RP) :: CQin_min_sum, CQin_min_sum_pl

  integer, parameter :: I_min = 1
  integer, parameter :: I_max = 2
  real(RP) :: Qin    (ADM_gall   ,ADM_kall,ADM_lall   ,2,6)
  real(RP) :: Qin_pl (ADM_gall_pl,ADM_kall,ADM_lall_pl,2,2)
  real(RP) :: Qout   (ADM_gall   ,ADM_kall,ADM_lall   ,2  )
  real(RP) :: Qout_pl(ADM_gall_pl,ADM_kall,ADM_lall_pl,2  )

  real(RP) :: ch_masked1
  real(RP) :: ch_masked2
  real(RP) :: ch_masked3
  real(RP) :: ch_masked4
  real(RP) :: ch_masked5
  real(RP) :: ch_masked6
  real(RP) :: ch_masked
  real(RP) :: zerosw

  integer  :: gmin, gmax, kall, iall
  real(RP) :: EPS, BIG

  integer  :: ij
  integer  :: ip1j, ijp1, ip1jp1, ip2jp1
  integer  :: im1j, ijm1

  integer  :: i, j, k, l, n, v
  !---------------------------------------------------------------------------

```

`q_a` is q at the edge of hexagonal control cell, which modified by the flux limiter. Note that `q_a` has 4-th dimension and its size is 6, that specifies 6 edges of hexagon control volume. `q` is q at grid point, `d` is a correction factor derived from an artificial viscosity for the total density. `ch` is Courant number, `cmask` is upwind direction mask. In original subroutine in NICAM, `Qout_prev` and `Qout_post` are not exist. These additional arguments is prepared to avoid halo communication, which appeared in the middle of this scheme. The detail is discussed below.

The first section of the subroutine is as follows.

```
  call DEBUG_rapstart('____horizontal_adv_limiter')

  gmin = ADM_gmin
  gmax = ADM_gmax
  kall = ADM_kall
  iall = ADM_gall_1d

  EPS  = CONST_EPS
  BIG  = CONST_HUGE

  do l = 1, ADM_lall
     !$omp parallel default(none), &
     !$omp private(i,j,k,ij,ip1j,ip1jp1,ijp1,im1j,ijm1,ip2jp1,                   &
     !$omp         q_min_AI,q_min_AIJ,q_min_AJ,q_max_AI,q_max_AIJ,q_max_AJ,zerosw,    &
     !$omp         ch_masked1,ch_masked2,ch_masked3,ch_masked4,ch_masked5,ch_masked6, &
     !$omp         qnext_min,qnext_max,Cin_sum,Cout_sum,CQin_min_sum,CQin_max_sum),   &
     !$omp shared(l,ADM_have_sgp,gmin,gmax,kall,iall,q,cmask,d,ch,Qin,Qout,EPS,BIG)
     do k = 1, kall
        !---< (i) define inflow bounds, eq.(32)&(33) >---
!OCL XFILL
        !$omp do
        do j = gmin-1, gmax
        do i = gmin-1, gmax
           ij     = (j-1)*iall + i
           ip1j   = ij + 1
           ip1jp1 = ij + iall + 1
           ijp1   = ij + iall
           im1j   = ij - 1
           ijm1   = ij - iall

           im1j   = max( im1j  , 1 )
           ijm1   = max( ijm1  , 1 )

           q_min_AI  = min( q(ij,k,l), q(ijm1,k,l), q(ip1j,k,l), q(ip1jp1,k,l) )
           q_max_AI  = max( q(ij,k,l), q(ijm1,k,l), q(ip1j,k,l), q(ip1jp1,k,l) )
           q_min_AIJ = min( q(ij,k,l), q(ip1j,k,l), q(ip1jp1,k,l), q(ijp1,k,l) )
           q_max_AIJ = max( q(ij,k,l), q(ip1j,k,l), q(ip1jp1,k,l), q(ijp1,k,l) )
```

```
118            q_min_AJ  = min( q(ij,k,l), q(ip1jp1,k,l), q(ijp1,k,l), q(im1j,k,l) )
119            q_max_AJ  = max( q(ij,k,l), q(ip1jp1,k,l), q(ijp1,k,l), q(im1j,k,l) )
120
121            Qin(ij,    k,l,I_min,1) = (        cmask(ij,k,l,1) ) * q_min_AI &
122                                     + ( 1.0_RP-cmask(ij,k,l,1) ) * BIG
123            Qin(ip1j,  k,l,I_min,4) = (        cmask(ij,k,l,1) ) * BIG      &
124                                     + ( 1.0_RP-cmask(ij,k,l,1) ) * q_min_AI
125            Qin(ij,    k,l,I_max,1) = (        cmask(ij,k,l,1) ) * q_max_AI &
126                                     + ( 1.0_RP-cmask(ij,k,l,1) ) * (-BIG)
127            Qin(ip1j,  k,l,I_max,4) = (        cmask(ij,k,l,1) ) * (-BIG)   &
128                                     + ( 1.0_RP-cmask(ij,k,l,1) ) * q_max_AI
129
130            Qin(ij,    k,l,I_min,2) = (        cmask(ij,k,l,2) ) * q_min_AIJ &
131                                     + ( 1.0_RP-cmask(ij,k,l,2) ) * BIG
132            Qin(ip1jp1,k,l,I_min,5) = (        cmask(ij,k,l,2) ) * BIG       &
133                                     + ( 1.0_RP-cmask(ij,k,l,2) ) * q_min_AIJ
134            Qin(ij,    k,l,I_max,2) = (        cmask(ij,k,l,2) ) * q_max_AIJ &
135                                     + ( 1.0_RP-cmask(ij,k,l,2) ) * (-BIG)
136            Qin(ip1jp1,k,l,I_max,5) = (        cmask(ij,k,l,2) ) * (-BIG)    &
137                                     + ( 1.0_RP-cmask(ij,k,l,2) ) * q_max_AIJ
138
139            Qin(ij,    k,l,I_min,3) = (        cmask(ij,k,l,3) ) * q_min_AJ &
140                                     + ( 1.0_RP-cmask(ij,k,l,3) ) * BIG
141            Qin(ijp1,  k,l,I_min,6) = (        cmask(ij,k,l,3) ) * BIG      &
142                                     + ( 1.0_RP-cmask(ij,k,l,3) ) * q_min_AJ
143            Qin(ij,    k,l,I_max,3) = (        cmask(ij,k,l,3) ) * q_max_AJ &
144                                     + ( 1.0_RP-cmask(ij,k,l,3) ) * (-BIG)
145            Qin(ijp1,  k,l,I_max,6) = (        cmask(ij,k,l,3) ) * (-BIG)   &
146                                     + ( 1.0_RP-cmask(ij,k,l,3) ) * q_max_AJ
147         enddo
148         enddo
149         !$omp end do
150
151         if ( ADM_have_sgp(l) ) then
152            !$omp master
153            j = gmin-1
154            i = gmin-1
155
156            ij     = (j-1)*iall + i
157            ijp1   = ij + iall
158            ip1jp1 = ij + iall + 1
159            ip2jp1 = ij + iall + 2
160
161            q_min_AIJ = min( q(ij,k,l), q(ip1jp1,k,l), q(ip2jp1,k,l), q(ijp1,k,l) )
162            q_max_AIJ = max( q(ij,k,l), q(ip1jp1,k,l), q(ip2jp1,k,l), q(ijp1,k,l) )
163
164            Qin(ij,    k,l,I_min,2) = (        cmask(ij,k,l,2) ) * q_min_AIJ &
165                                     + ( 1.0_RP-cmask(ij,k,l,2) ) * BIG
166            Qin(ip1jp1,k,l,I_min,5) = (        cmask(ij,k,l,2) ) * BIG       &
167                                     + ( 1.0_RP-cmask(ij,k,l,2) ) * q_min_AIJ
168            Qin(ij,    k,l,I_max,2) = (        cmask(ij,k,l,2) ) * q_max_AIJ &
169                                     + ( 1.0_RP-cmask(ij,k,l,2) ) * (-BIG)
170            Qin(ip1jp1,k,l,I_max,5) = (        cmask(ij,k,l,2) ) * (-BIG)    &
171                                     + ( 1.0_RP-cmask(ij,k,l,2) ) * q_max_AIJ
172            !$omp end master
173         endif
174
```

The first section above and second section below are in long l- and k- double loop. In that there are 2 main i,j-double loops and one `IF` clause and a small i,j-loop. The i,j-loop(l.102) calculates the inflow bounds. First, minimum and maximum of q at arc points (specified by `AI`, `AIJ` and `AJ`) are set, then actual bound of inflow `Qin` is calcurated. `IF` clause at l.151 is a treatment for the singular points.

Second section is as follows.

```
175         !---< (iii) define allowable range of q at next step, eq.(42)&(43) >---
176 !OCL XFILL
177         !$omp do
178         do j = gmin, gmax
179         do i = gmin, gmax
180            ij = (j-1)*iall + i
181
182            qnext_min = min( q(ij,k,l),          &
183                             Qin(ij,k,l,I_min,1), &
184                             Qin(ij,k,l,I_min,2), &
185                             Qin(ij,k,l,I_min,3), &
186                             Qin(ij,k,l,I_min,4), &
187                             Qin(ij,k,l,I_min,5), &
188                             Qin(ij,k,l,I_min,6)  )
```

```fortran

            qnext_max = max( q(ij,k,l),            &
                             Qin(ij,k,l,I_max,1), &
                             Qin(ij,k,l,I_max,2), &
                             Qin(ij,k,l,I_max,3), &
                             Qin(ij,k,l,I_max,4), &
                             Qin(ij,k,l,I_max,5), &
                             Qin(ij,k,l,I_max,6)  )

            ch_masked1 = min( ch(ij,k,l,1), 0.0_RP )
            ch_masked2 = min( ch(ij,k,l,2), 0.0_RP )
            ch_masked3 = min( ch(ij,k,l,3), 0.0_RP )
            ch_masked4 = min( ch(ij,k,l,4), 0.0_RP )
            ch_masked5 = min( ch(ij,k,l,5), 0.0_RP )
            ch_masked6 = min( ch(ij,k,l,6), 0.0_RP )

            Cin_sum      = ch_masked1 &
                         + ch_masked2 &
                         + ch_masked3 &
                         + ch_masked4 &
                         + ch_masked5 &
                         + ch_masked6

            Cout_sum     = ch(ij,k,l,1) - ch_masked1 &
                         + ch(ij,k,l,2) - ch_masked2 &
                         + ch(ij,k,l,3) - ch_masked3 &
                         + ch(ij,k,l,4) - ch_masked4 &
                         + ch(ij,k,l,5) - ch_masked5 &
                         + ch(ij,k,l,6) - ch_masked6

            CQin_min_sum = ch_masked1 * Qin(ij,k,l,I_min,1) &
                         + ch_masked2 * Qin(ij,k,l,I_min,2) &
                         + ch_masked3 * Qin(ij,k,l,I_min,3) &
                         + ch_masked4 * Qin(ij,k,l,I_min,4) &
                         + ch_masked5 * Qin(ij,k,l,I_min,5) &
                         + ch_masked6 * Qin(ij,k,l,I_min,6)

            CQin_max_sum = ch_masked1 * Qin(ij,k,l,I_max,1) &
                         + ch_masked2 * Qin(ij,k,l,I_max,2) &
                         + ch_masked3 * Qin(ij,k,l,I_max,3) &
                         + ch_masked4 * Qin(ij,k,l,I_max,4) &
                         + ch_masked5 * Qin(ij,k,l,I_max,5) &
                         + ch_masked6 * Qin(ij,k,l,I_max,6)

            zerosw = 0.5_RP - sign(0.5_RP,abs(Cout_sum)-EPS) ! if Cout_sum = 0, sw = 1

            Qout(ij,k,l,I_min) = ( q(ij,k,l) - CQin_max_sum - qnext_max*(1.0_RP-Cin_sum-Cout_sum+d(ij,k,l)) ) &
                               / ( Cout_sum + zerosw ) * ( 1.0_RP - zerosw )                                  &
                               + q(ij,k,l) * zerosw
            Qout(ij,k,l,I_max) = ( q(ij,k,l) - CQin_min_sum - qnext_min*(1.0_RP-Cin_sum-Cout_sum+d(ij,k,l)) ) &
                               / ( Cout_sum + zerosw ) * ( 1.0_RP - zerosw )                                  &
                               + q(ij,k,l) * zerosw
         enddo
         enddo
         !$omp end do

!OCL XFILL
         !$omp do
         do j = 1, iall
         do i = 1, iall
            if (      i < gmin .OR. i > gmax &
                 .OR. j < gmin .OR. j > gmax ) then
               ij = (j-1)*iall + i

               Qout(ij,k,l,I_min) = q(ij,k,l)
               Qout(ij,k,l,I_min) = q(ij,k,l)
               Qout(ij,k,l,I_max) = q(ij,k,l)
               Qout(ij,k,l,I_max) = q(ij,k,l)
            endif
         enddo
         enddo
         !$omp end do

      enddo ! k loop
      !$omp end parallel
   enddo ! l loop

```

There are two i, j-double loops. In the first one, allowable range of q is defined as `qnext_min`, `qnext_max`, then `Qout` is calculated. And the second double loop is to set halo region.

The third section is as follows. This section is for the pole region, doing almost the same calculation with the normal region.

```
   if ( ADM_have_pl ) then
      n = ADM_gslf_pl

      do l = 1, ADM_lall_pl
         do k = 1, ADM_kall
            do v = ADM_gmin_pl, ADM_gmax_pl
               ij   = v
               ijp1 = v + 1
               ijm1 = v - 1
               if( ijp1 == ADM_gmax_pl+1 ) ijp1 = ADM_gmin_pl
               if( ijm1 == ADM_gmin_pl-1 ) ijm1 = ADM_gmax_pl

               q_min_pl = min( q_pl(n,k,l), q_pl(ij,k,l), q_pl(ijm1,k,l), q_pl(ijp1,k,l) )
               q_max_pl = max( q_pl(n,k,l), q_pl(ij,k,l), q_pl(ijm1,k,l), q_pl(ijp1,k,l) )

               Qin_pl(ij,k,l,I_min,1) = (        cmask_pl(ij,k,l) ) * q_min_pl &
                                      + ( 1.0_RP-cmask_pl(ij,k,l) ) * BIG
               Qin_pl(ij,k,l,I_min,2) = (        cmask_pl(ij,k,l) ) * BIG      &
                                      + ( 1.0_RP-cmask_pl(ij,k,l) ) * q_min_pl
               Qin_pl(ij,k,l,I_max,1) = (        cmask_pl(ij,k,l) ) * q_max_pl &
                                      + ( 1.0_RP-cmask_pl(ij,k,l) ) * (-BIG)
               Qin_pl(ij,k,l,I_max,2) = (        cmask_pl(ij,k,l) ) * (-BIG)   &
                                      + ( 1.0_RP-cmask_pl(ij,k,l) ) * q_max_pl
            enddo

            qnext_min_pl = q_pl(n,k,l)
            qnext_max_pl = q_pl(n,k,l)
            do v = ADM_gmin_pl, ADM_gmax_pl
               qnext_min_pl = min( qnext_min_pl, Qin_pl(v,k,l,I_min,1) )
               qnext_max_pl = max( qnext_max_pl, Qin_pl(v,k,l,I_max,1) )
            enddo

            Cin_sum_pl      = 0.0_RP
            Cout_sum_pl     = 0.0_RP
            CQin_max_sum_pl = 0.0_RP
            CQin_min_sum_pl = 0.0_RP
            do v = ADM_gmin_pl, ADM_gmax_pl
               ch_masked = cmask_pl(v,k,l) * ch_pl(v,k,l)

               Cin_sum_pl      = Cin_sum_pl      + ch_masked
               Cout_sum_pl     = Cout_sum_pl     - ch_masked + ch_pl (v,k,l)
               CQin_min_sum_pl = CQin_min_sum_pl + ch_masked * Qin_pl(v,k,l,I_min,1)
               CQin_max_sum_pl = CQin_max_sum_pl + ch_masked * Qin_pl(v,k,l,I_max,1)
            enddo

            zerosw = 0.5_RP - sign(0.5_RP,abs(Cout_sum_pl)-EPS) ! if Cout_sum_pl = 0, sw = 1

            Qout_pl(n,k,l,I_min) = ( q_pl(n,k,l) - CQin_max_sum_pl - qnext_max_pl*(1.0_RP-Cin_sum_pl-Cout_sum_pl+d_pl(n,k,l)) ) &
                                 / ( Cout_sum_pl + zerosw ) * ( 1.0_RP - zerosw )                                              &
                                 + q_pl(n,k,l) * zerosw
            Qout_pl(n,k,l,I_max) = ( q_pl(n,k,l) - CQin_min_sum_pl - qnext_min_pl*(1.0_RP-Cin_sum_pl-Cout_sum_pl+d_pl(n,k,l)) ) &
                                 / ( Cout_sum_pl + zerosw ) * ( 1.0_RP - zerosw )                                              &
                                 + q_pl(n,k,l) * zerosw

         enddo
      enddo
   endif

```

In the original subroutine, halo exchange using communication subroutine `COMM_data_transfer` here. In kernelization process, this comunication is omitted. Values to be sent is output for the purpose of validation, and values to be received are given as an argument, read from input data file in the main program of this kernel program.

```
   !#########################################################################KERNEL
   call DEBUG_rapend  ('____horizontal_adv_limiter')
   Qout_pl(ADM_gmin_pl:ADM_gmax_pl,:,:,:) = 0.0_RP

   Qout_prev   (:,:,:,:) = Qout        (:,:,:,:)
   Qout_prev_pl(:,:,:,:) = Qout_pl     (:,:,:,:)
   !call COMM_data_transfer( Qout(:,:,:,:), Qout_pl(:,:,:,:) )
   Qout        (:,:,:,:) = Qout_post   (:,:,:,:)
   Qout_pl     (:,:,:,:) = Qout_post_pl(:,:,:,:)
   call DEBUG_rapstart('____horizontal_adv_limiter')
```

```
    !#########################################################################KERNEL

```

The next section is as follows.

```
    !---- apply inflow/outflow limiter
    do l = 1, ADM_lall
       !$omp parallel do default(none),private(i,j,k,ij,ip1j,ip1jp1,ijp1), &
       !$omp shared(l,gmin,gmax,kall,iall,q_a,cmask,Qin,Qout)
       do k = 1, kall
          do j = gmin-1, gmax
          do i = gmin-1, gmax
             ij     = (j-1)*iall + i
             ip1j   = ij + 1
             ip1jp1 = ij + iall + 1
             ijp1   = ij + iall

             q_a(ij,k,l,1) = (        cmask(ij,k,l,1) ) * min( max( q_a(ij,k,l,1), Qin (ij    ,k,l,I_min,1) ), Qin (ij    ,k,l,I_max,1) ) &
                           + ( 1.0_RP-cmask(ij,k,l,1) ) * min( max( q_a(ij,k,l,1), Qin (ip1j  ,k,l,I_min,4) ), Qin (ip1j  ,k,l,I_max,4) )
             q_a(ij,k,l,1) = (        cmask(ij,k,l,1) ) * max( min( q_a(ij,k,l,1), Qout(ip1j  ,k,l,I_max  ) ), Qout(ip1j  ,k,l,I_min  ) ) &
                           + ( 1.0_RP-cmask(ij,k,l,1) ) * max( min( q_a(ij,k,l,1), Qout(ij    ,k,l,I_max  ) ), Qout(ij    ,k,l,I_min  ) )
             q_a(ip1j,k,l,4) = q_a(ij,k,l,1)

             q_a(ij,k,l,2) = (        cmask(ij,k,l,2) ) * min( max( q_a(ij,k,l,2), Qin (ij    ,k,l,I_min,2) ), Qin (ij    ,k,l,I_max,2) ) &
                           + ( 1.0_RP-cmask(ij,k,l,2) ) * min( max( q_a(ij,k,l,2), Qin (ip1jp1,k,l,I_min,5) ), Qin (ip1jp1,k,l,I_max,5) )
             q_a(ij,k,l,2) = (        cmask(ij,k,l,2) ) * max( min( q_a(ij,k,l,2), Qout(ip1jp1,k,l,I_max  ) ), Qout(ip1jp1,k,l,I_min  ) ) &
                           + ( 1.0_RP-cmask(ij,k,l,2) ) * max( min( q_a(ij,k,l,2), Qout(ij    ,k,l,I_max  ) ), Qout(ij    ,k,l,I_min  ) )
             q_a(ip1jp1,k,l,5) = q_a(ij,k,l,2)

             q_a(ij,k,l,3) = (        cmask(ij,k,l,3) ) * min( max( q_a(ij,k,l,3), Qin (ij    ,k,l,I_min,3) ), Qin (ij    ,k,l,I_max,3) ) &
                           + ( 1.0_RP-cmask(ij,k,l,3) ) * min( max( q_a(ij,k,l,3), Qin (ijp1  ,k,l,I_min,6) ), Qin (ijp1  ,k,l,I_max,6) )
             q_a(ij,k,l,3) = (        cmask(ij,k,l,3) ) * max( min( q_a(ij,k,l,3), Qout(ijp1  ,k,l,I_max  ) ), Qout(ijp1  ,k,l,I_min  ) ) &
                           + ( 1.0_RP-cmask(ij,k,l,3) ) * max( min( q_a(ij,k,l,3), Qout(ij    ,k,l,I_max  ) ), Qout(ij    ,k,l,I_min  ) )
             q_a(ijp1,k,l,6) = q_a(ij,k,l,3)
          enddo
          enddo
       enddo
       !$omp end parallel do
    enddo

```

In new l- and k- double loop, there is a i, j-double loop, calculating limitter apllied tracer quantities at each cell faces, i.e. edges of the hexagonal control volume. Note that only 1st, 2nd, 3rd edge must be calculated, because other 3 edges' value is the same with other grid's 1st, 2nd, or 3rd edge.

The last part of this subroutine is as follows.

```
    if ( ADM_have_pl ) then
       n = ADM_gslf_pl

       do l = 1, ADM_lall_pl
       do k = 1, ADM_kall
       do v = ADM_gmin_pl, ADM_gmax_pl
          q_a_pl(v,k,l) = (        cmask_pl(v,k,l) ) * min(max(q_a_pl(v,k,l), Qin_pl (v,k,l,I_min,1)), Qin_pl (v,k,l,I_max,1)) &
                        + ( 1.0_RP-cmask_pl(v,k,l) ) * min(max(q_a_pl(v,k,l), Qin_pl (v,k,l,I_min,2)), Qin_pl (v,k,l,I_max,2))
          q_a_pl(v,k,l) = (        cmask_pl(v,k,l) ) * max(min(q_a_pl(v,k,l), Qout_pl(v,k,l,I_max  )), Qout_pl(v,k,l,I_min  )) &
                        + ( 1.0_RP-cmask_pl(v,k,l) ) * max(min(q_a_pl(v,k,l), Qout_pl(n,k,l,I_max  )), Qout_pl(n,k,l,I_min  ))
       enddo
       enddo
       enddo
    endif

    call DEBUG_rapend  ('____horizontal_adv_limiter')

    return
  end subroutine horizontal_limiter_thuburn
```

In this part, `q_a` in the pole region are calculated, with almost the same procedure with the normal region.

2.7.3 Input data and result

Max/min/sum of input/output data of the kernel subroutine are output as a log. Below is an example of `$IAB_SYS=Ubuntu-gnu-ompi` case.

```
### Input ###
+check[q_a_prev        ] max=  3.1443592333952495E+02,min= -3.3659800625927818E+02,sum=  1.4867286349717416E+07
+check[q_a_prev_pl     ] max=  7.5768030875957253E+00,min= -5.6211573453294959E-15,sum=  1.5762773946695211E+03
+check[check_q_a       ] max=  3.0706553064103554E+02,min= -2.3487181740471829E+02,sum=  1.4857441189951191E+07
+check[check_q_a_pl    ] max=  6.5571419615446418E+00,min= -5.6211573453294959E-15,sum=  1.5683664645246540E+03
+check[q               ] max=  7.3394566876316425E+00,min=  0.0000000000000000E+00,sum=  2.4959622235983950E+06
+check[q_pl            ] max=  6.5571419615446418E+00,min=  0.0000000000000000E+00,sum=  1.8536812127383619E+03
+check[d               ] max=  1.7307542155642468E-05,min= -1.9433424502805651E-05,sum=  6.2175140984963743E-06
+check[d_pl            ] max=  6.2231074370491122E-06,min= -2.0788320527281462E-05,sum= -1.1073365231406677E-03
+check[ch              ] max=  2.5115202433354633E-01,min= -2.8726813406847629E-01,sum= -6.8752454869001423E-01
+check[ch_pl           ] max=  1.9806049765030476E-01,min= -1.2208152245920290E+01,sum= -3.1022849107616275E+02
+check[cmask           ] max=  1.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  2.1729060000000000E+06
+check[cmask_pl        ] max=  1.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  2.1800000000000000E+02
+check[check_Qout_prev ] max=  2.5201719369799781E+05,min= -7.0121044696269173E+05,sum=  4.7462261170534119E+06
+check[check_Qout_prev_] max=  3.4849498839973911E+01,min= -3.6368061161685844E+01,sum=  7.2956220495347202E+02
+check[Qout_post       ] max=  2.5201719369799781E+05,min= -7.0121044696269173E+05,sum=  4.7387405453230906E+06
+check[Qout_post_pl    ] max=  5.9738451374182347E+01,min= -3.6368061161685844E+01,sum=  4.0063169963931473E+03
### Output ###
+check[q_a             ] max=  3.0706553064103554E+02,min= -2.3487181740471829E+02,sum=  1.4857441189951191E+07
+check[q_a_pl          ] max=  6.5571419615446418E+00,min= -5.6211573453294959E-15,sum=  1.5683664645246540E+03
+check[Qout_prev       ] max=  2.5201719369799781E+05,min= -7.0121044696269173E+05,sum=  4.7462261170534119E+06
+check[Qout_prev_pl    ] max=  3.4849498839973911E+01,min= -3.6368061161685844E+01,sum=  7.2956220495347202E+02
### Validation : point-by-point diff ###
+check[check_q_a       ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_q_a_pl    ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_Qout_prev ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
+check[check_Qout_prev_] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
*** Finish kernel
```

Check the lines below "`Validation : point-by-point diff`" line, that shows difference between calculated output array and pre-calculated reference array. These should be zero or enough small to be acceptable.

There are sample output log files in `reference/` in each kernel program directory, for reference purpose.

2.7.4 Sample of perfomance result

Here's an example of the performance result part of the log output. Below is an example executed with the machine environment described in subsection 2.1.7. Note that in this program kernel part is iterated one time.

```
*** Computational Time Report
*** ID=001 : MAIN_dyn_horiz_adv_limiter       T=     0.180 N=      1
*** ID=002 : ____horizontal_adv_limiter       T=     0.164 N=      2
```

2.8 `dyn_metrics`

2.8.1 Description

Kernel `dyn_metrics` gathers calcuration part of various metric terms in several subroutines listed below:

- `GMTR_p_setup`
- `GMTR_t_setup`
- `GMTR_a_setup`
- `OPRT_divergence_setup`
- `OPRT_rotation_setup`
- `OPRT_gradient_setup`
- `OPRT_laplacian_setup`
- `OPRT_diffusion_setup`

These subroutines are defined as the same name in module `mod_gmtr` and `mod_oprt` in original *NICAM*.

2.8.2 Discretization and code

(1) `GMTR_p_setup`

`mod_gmtr` contains public objects related to the metrics. This subroutine is to setup metrics for the cell point.

Argument lists and local variables definition part of this subroutine is as follows.

```
subroutine GMTR_p_setup( &
     GRD_x,   GRD_x_pl,   &
     GRD_xt,  GRD_xt_pl,  &
     GRD_s,   GRD_s_pl,   &
     GMTR_p,  GMTR_p_pl,  &
     GRD_rscale           )
!ESC!    use mod_adm, only: &
!ESC!       ADM_nxyz,      &
!ESC!       ADM_have_pl,   &
!ESC!       ADM_have_sgp,  &
!ESC!       ADM_vlink,     &
!ESC!       ADM_gmin,      &
!ESC!       ADM_gmax,      &
!ESC!       ADM_gslf_pl
!ESC!    use mod_grd, only: &
!ESC!       GRD_XDIR,               &
!ESC!       GRD_YDIR,               &
!ESC!       GRD_ZDIR,               &
!ESC!       GRD_LON,                &
!ESC!       GRD_grid_type_on_plane, &
!ESC!       GRD_grid_type
  use mod_vector, only: &
     VECTR_triangle,      &
     VECTR_triangle_plane
  implicit none

  real(RP), intent(in)  :: GRD_x     (ADM_gall   ,k0,ADM_lall   ,      ADM_nxyz)
  real(RP), intent(in)  :: GRD_x_pl  (ADM_gall_pl,k0,ADM_lall_pl,      ADM_nxyz)
  real(RP), intent(in)  :: GRD_xt    (ADM_gall   ,k0,ADM_lall   ,TI:TJ,ADM_nxyz)
  real(RP), intent(in)  :: GRD_xt_pl(ADM_gall_pl,k0,ADM_lall_pl,       ADM_nxyz)
  real(RP), intent(in)  :: GRD_s     (ADM_gall   ,k0,ADM_lall   ,      2)
  real(RP), intent(in)  :: GRD_s_pl  (ADM_gall_pl,k0,ADM_lall_pl,      2)
  real(RP), intent(out) :: GMTR_p    (ADM_gall   ,k0,ADM_lall   ,GMTR_p_nmax)
  real(RP), intent(out) :: GMTR_p_pl(ADM_gall_pl,k0,ADM_lall_pl,GMTR_p_nmax)
  real(RP), intent(in)  :: GRD_rscale

  real(RP) :: wk    (ADM_nxyz,0:7,ADM_gall)
  real(RP) :: wk_pl(ADM_nxyz,0:ADM_vlink+1)

  real(RP) :: area
  real(RP) :: cos_lambda, sin_lambda

  integer  :: ij
  integer  :: ip1j, ijp1, ip1jp1
  integer  :: im1j, ijm1, im1jm1

  integer  :: i, j, l, d, v, n
  !-------------------------------------------------------------------------

```

`GRD_x` and `GRD_xt` are the coordinates in 3-D Cartesian of the center points and the vertex points of control cell, respectively. And `GRD_s` is the coordinates in the spherical coordinate on the planet i.e. latitude and longitude in radian. Those with suffix `_pl` are for the pole region. These coodinate values are given as arguments, read from input data file in this kernel prograum. `GMTR_p` and `GMTR_p_pl` are the metrics this subroutine calculates for normal region and pole region, respectively. The last dimension of these have the size of `GMTR_p_nmax`, each element of this dimension specify the kind of various metrics. For example, `GMTR_p(:,:,:,GMTR_p_AREA)` means area of the hexagonal controll cell `GRD_rscale` is a scaling factor for the radius of the sphere, and set as `6.37122E+6_RP` [m] for this kernel program in `problem_size.inc`.

There is one long l-loop in this subroutine, divided to 3 sections. The first section is as follows.

```
  !if( IO_L ) write(IO_FID_LOG,*) '*** setup metrics for hexagonal/pentagonal mesh'

  GMTR_p    (:,:,:,:)   = 0.0_RP
  GMTR_p_pl(:,:,:,:)    = 0.0_RP

  do l = 1, ADM_lall
```

```
56       do j = ADM_gmin, ADM_gmax
57       do i = ADM_gmin, ADM_gmax
58          ij     = suf(i  ,j  )
59          ip1j   = suf(i+1,j  )
60          ip1jp1 = suf(i+1,j+1)
61          ijp1   = suf(i  ,j+1)
62          im1j   = suf(i-1,j  )
63          im1jm1 = suf(i-1,j-1)
64          ijm1   = suf(i  ,j-1)
65
66          !--- prepare 1 center and 6 vertices
67          do d = 1, ADM_nxyz
68             wk(d,0,ij) = GRD_x(ij,k0,l,d)
69
70             wk(d,1,ij) = GRD_xt(ijm1  ,k0,l,TJ,d)
71             wk(d,2,ij) = GRD_xt(ij    ,k0,l,TI,d)
72             wk(d,3,ij) = GRD_xt(ij    ,k0,l,TJ,d)
73             wk(d,4,ij) = GRD_xt(im1j  ,k0,l,TI,d)
74             wk(d,5,ij) = GRD_xt(im1jm1,k0,l,TJ,d)
75             wk(d,6,ij) = GRD_xt(im1jm1,k0,l,TI,d)
76             wk(d,7,ij) = wk(d,1,ij)
77          enddo
78       enddo ! i loop
79       enddo ! j loop
80
81       if ( ADM_have_sgp(l) ) then ! pentagon
82          wk(:,6,suf(ADM_gmin,ADM_gmin)) = wk(:,1,suf(ADM_gmin,ADM_gmin))
83          wk(:,7,suf(ADM_gmin,ADM_gmin)) = wk(:,1,suf(ADM_gmin,ADM_gmin))
84       endif
85
```

In this first section, the coordinates of center and 6 vertices of a control volume are set to a temporary array `wk`. `ADM_nxyz` is 3 and the inner-most loop index `d` specifies (X, Y, Z) direction of 3-D Cartesian coodinates. As for the last `IF` clause(l.81), if `ADM_have_sgp(l)` is true, the l-th region has the singular point and the control volume of that point is not a hexagon but a pentagon.

The second section of the main l-loop is as follows.

```
86       !--- calc control area
87       if ( GRD_grid_type == GRD_grid_type_on_plane ) then
88          do j = ADM_gmin, ADM_gmax
89          do i = ADM_gmin, ADM_gmax
90             ij = suf(i,j)
91
92             area = 0.0_RP
93             do v = 1, 6
94                area = area + VECTR_triangle_plane( wk(:,0,ij), wk(:,v,ij), wk(:,v+1,ij) )
95             enddo
96
97             GMTR_p(ij,k0,l,GMTR_p_AREA)  = area
98             GMTR_p(ij,k0,l,GMTR_p_RAREA) = 1.0_RP / GMTR_p(ij,k0,l,GMTR_p_AREA)
99
100         enddo ! i loop
101         enddo ! j loop
102      else
103         do j = ADM_gmin, ADM_gmax
104         do i = ADM_gmin, ADM_gmax
105            ij = suf(i,j)
106
107            wk(:,:,ij) = wk(:,:,ij) / GRD_rscale
108
109            area = 0.0_RP
110            do v = 1, 6
111               area = area + VECTR_triangle( wk(:,0,ij), wk(:,v,ij), wk(:,v+1,ij), GMTR_polygon_type, GRD_rscale )
112            enddo
113
114            GMTR_p(ij,k0,l,GMTR_p_AREA)  = area
115            GMTR_p(ij,k0,l,GMTR_p_RAREA) = 1.0_RP / GMTR_p(ij,k0,l,GMTR_p_AREA)
116
117         enddo ! i loop
118         enddo ! j loop
119      endif
120
```

This section calculates the area size of control volume, by summing up 6 triangles that consist of the hexagon. `GMTR_p(:,:,:,GMTR_p_RAREA)` is a reciprocal of the size of area.

The last part of this main l-loop is as follows.

```
      !--- calc coefficient between xyz <-> latlon
      if ( GRD_grid_type == GRD_grid_type_on_plane ) then
         GMTR_p(:,k0,l,GMTR_p_IX) = 1.0_RP
         GMTR_p(:,k0,l,GMTR_p_IY) = 0.0_RP
         GMTR_p(:,k0,l,GMTR_p_IZ) = 0.0_RP
         GMTR_p(:,k0,l,GMTR_p_JX) = 0.0_RP
         GMTR_p(:,k0,l,GMTR_p_JY) = 1.0_RP
         GMTR_p(:,k0,l,GMTR_p_JZ) = 0.0_RP
      else
         do j = ADM_gmin, ADM_gmax
         do i = ADM_gmin, ADM_gmax
            ij = suf(i,j)

            sin_lambda = sin( GRD_s(ij,k0,l,GRD_LON) )
            cos_lambda = cos( GRD_s(ij,k0,l,GRD_LON) )

            GMTR_p(ij,k0,l,GMTR_p_IX) = -sin_lambda
            GMTR_p(ij,k0,l,GMTR_p_IY) =  cos_lambda
            GMTR_p(ij,k0,l,GMTR_p_IZ) = 0.0_RP
            GMTR_p(ij,k0,l,GMTR_p_JX) = -( GRD_x(ij,k0,l,ZDIR) * cos_lambda ) / GRD_rscale
            GMTR_p(ij,k0,l,GMTR_p_JY) = -( GRD_x(ij,k0,l,ZDIR) * sin_lambda ) / GRD_rscale
            GMTR_p(ij,k0,l,GMTR_p_JZ) =  ( GRD_x(ij,k0,l,XDIR) * cos_lambda &
                                        + GRD_x(ij,k0,l,YDIR) * sin_lambda ) / GRD_rscale
         enddo ! i loop
         enddo ! j loop
      endif
   enddo ! l loop

```

This section calculates the coefficients used by conversion from 3-D Cartesian coordinates to the spherical coordinates and *vice versa*. Note that `GRD_grid_type == GRD_grid_type_on_plane` is set as false for this kernel program in `problem_size.inc`.

The remaining part of this subroutine is as follows.

```
   if ( ADM_have_pl ) then
      n = ADM_gslf_pl

      do l = 1, ADM_lall_pl
         !--- prepare 1 center and * vertices
         do d = 1, ADM_nxyz
            wk_pl(d,0) = GRD_x_pl(n,k0,l,d)
            do v = 1, ADM_vlink ! (ICO=5)
               wk_pl(d,v) = GRD_xt_pl(v+1,k0,l,d)
            enddo
            wk_pl(d,ADM_vlink+1) = wk_pl(d,1)
         enddo

         wk_pl(:,:) = wk_pl(:,:) / GRD_rscale

         !--- calc control area
         area = 0.0_RP
         do v = 1, ADM_vlink ! (ICO=5)
            area = area + VECTR_triangle( wk_pl(:,0), wk_pl(:,v), wk_pl(:,v+1), GMTR_polygon_type, GRD_rscale )
         enddo

         GMTR_p_pl(n,k0,l,GMTR_p_AREA)  = area
         GMTR_p_pl(n,k0,l,GMTR_p_RAREA) = 1.0_RP / GMTR_p_pl(n,k0,l,GMTR_p_AREA)

         !--- calc coefficient between xyz <-> latlon
         sin_lambda = sin( GRD_s_pl(n,k0,l,GRD_LON) )
         cos_lambda = cos( GRD_s_pl(n,k0,l,GRD_LON) )

         GMTR_p_pl(n,k0,l,GMTR_p_IX) = -sin_lambda
         GMTR_p_pl(n,k0,l,GMTR_p_IY) =  cos_lambda
         GMTR_p_pl(n,k0,l,GMTR_p_IZ) = 0.0_RP
         GMTR_p_pl(n,k0,l,GMTR_p_JX) = -( GRD_x_pl(n,k0,l,ZDIR) * cos_lambda ) / GRD_rscale
         GMTR_p_pl(n,k0,l,GMTR_p_JY) = -( GRD_x_pl(n,k0,l,ZDIR) * sin_lambda ) / GRD_rscale
         GMTR_p_pl(n,k0,l,GMTR_p_JZ) =  ( GRD_x_pl(n,k0,l,XDIR) * cos_lambda &
                                       + GRD_x_pl(n,k0,l,YDIR) * sin_lambda ) / GRD_rscale
      enddo ! l loop
   endif

   return
end subroutine GMTR_p_setup
```

This part is to calculates for the pole region. Note that the pole region is a pentagon.

(2) GMTR_t_setup

This subroutine is to setup metrics for the cell vertices or the triangles.

Argument lists and local variables definition part of this subroutine is as follows.

```
subroutine GMTR_t_setup( &
     GRD_x,   GRD_x_pl,  &
     GRD_xt,  GRD_xt_pl, &
     GMTR_t,  GMTR_t_pl, &
     GRD_rscale          )
!ESC!    use mod_adm, only: &
!ESC!       ADM_nxyz,      &
!ESC!       ADM_have_pl,   &
!ESC!       ADM_have_sgp,  &
!ESC!       ADM_gmin,      &
!ESC!       ADM_gmax,      &
!ESC!       ADM_gslf_pl,   &
!ESC!       ADM_gmin_pl,   &
!ESC!       ADM_gmax_pl
!ESC!    use mod_grd, only: &
!ESC!       GRD_grid_type_on_plane, &
!ESC!       GRD_grid_type
  use mod_vector, only: &
     VECTR_triangle,      &
     VECTR_triangle_plane
  implicit none

  real(RP), intent(in)  :: GRD_x    (ADM_gall   ,k0,ADM_lall   ,              ADM_nxyz)
  real(RP), intent(in)  :: GRD_x_pl (ADM_gall_pl,k0,ADM_lall_pl,              ADM_nxyz)
  real(RP), intent(in)  :: GRD_xt   (ADM_gall   ,k0,ADM_lall   ,TI:TJ,ADM_nxyz)
  real(RP), intent(in)  :: GRD_xt_pl(ADM_gall_pl,k0,ADM_lall_pl,              ADM_nxyz)
  real(RP), intent(out) :: GMTR_t   (ADM_gall   ,k0,ADM_lall   ,TI:TJ,GMTR_t_nmax)
  real(RP), intent(out) :: GMTR_t_pl(ADM_gall_pl,k0,ADM_lall_pl,              GMTR_t_nmax)
  real(RP), intent(in)  :: GRD_rscale

  real(RP) :: wk   (ADM_nxyz,0:3,ADM_gall,TI:TJ)
  real(RP) :: wk_pl(ADM_nxyz,0:3)

  real(RP) :: area, area1, area2, area3

  integer  :: ij
  integer  :: ip1j, ijp1, ip1jp1

  integer  :: i, j, l, d, v, n, t
  !---------------------------------------------------------------------------

```

Input arguments are the same with the previous GMTR_p_setup. GMTR_t and GMTR_t_pl are the metrics for the vertex points of control cell for the normal region and the pole region, respectively. In the last dimension of these arrays whose size is GMTR_t_nmax, each element specifies the kind of various metrics. For example, GMTR_t(:,:,:,:,GMTR_t_AREA) means the area of the upward and downward triangle, which contains vertex points of control cell as the gravitational center.

Main part of this subroutine is a single l-loop, divided to two sections. The first section is as follows.

```
  !if( IO_L ) write(IO_FID_LOG,*) '*** setup metrics for triangle mesh'

  GMTR_t   (:,:,:,:,:) = 0.0_RP
  GMTR_t_pl(:,:,:,:)   = 0.0_RP

  do l = 1,ADM_lall
     do j = ADM_gmin-1, ADM_gmax
     do i = ADM_gmin-1, ADM_gmax
        ij     = suf(i  ,j  )
        ip1j   = suf(i+1,j  )
        ip1jp1 = suf(i+1,j+1)
        ijp1   = suf(i  ,j+1)

        !--- prepare 1 center and 3 vertices for 2 triangles
        do d = 1, ADM_nxyz
           wk(d,0,ij,TI) = GRD_xt(ij,k0,l,TI,d)

           wk(d,1,ij,TI) = GRD_x(ij    ,k0,l,d)
```

```
            wk(d,2,ij,TI) = GRD_x(ip1j  ,k0,l,d)
            wk(d,3,ij,TI) = GRD_x(ip1jp1,k0,l,d)

            wk(d,0,ij,TJ) = GRD_xt(ij,k0,l,TJ,d)

            wk(d,1,ij,TJ) = GRD_x(ij    ,k0,l,d)
            wk(d,2,ij,TJ) = GRD_x(ip1jp1,k0,l,d)
            wk(d,3,ij,TJ) = GRD_x(ijp1  ,k0,l,d)
         enddo
      enddo
      enddo

```

In this section, coordinates of the center and three vertices for two triangles represents. The meaning of `ADM_nxyz` and the loop index `d` are the same with them in the previous subroutine `GMTR_p_setup`.

The second section is as follows.

```
      !--- treat unused triangle
      wk(:,:,suf(ADM_gmax,ADM_gmin-1),TI) = wk(:,:,suf(ADM_gmax,ADM_gmin-1),TJ)
      wk(:,:,suf(ADM_gmin-1,ADM_gmax),TJ) = wk(:,:,suf(ADM_gmin-1,ADM_gmax),TI)

      if ( ADM_have_sgp(l) ) then ! pentagon
         wk(:,:,suf(ADM_gmin-1,ADM_gmin-1),TI) = wk(:,:,suf(ADM_gmin,ADM_gmin-1),TJ)
      endif

      if ( GRD_grid_type == GRD_grid_type_on_plane ) then
         do t = TI,TJ
         do j = ADM_gmin-1, ADM_gmax
         do i = ADM_gmin-1, ADM_gmax
            ij = suf(i,j)

            area1 = VECTR_triangle_plane( wk(:,0,ij,t), wk(:,2,ij,t), wk(:,3,ij,t) )
            area2 = VECTR_triangle_plane( wk(:,0,ij,t), wk(:,3,ij,t), wk(:,1,ij,t) )
            area3 = VECTR_triangle_plane( wk(:,0,ij,t), wk(:,1,ij,t), wk(:,2,ij,t) )

            area = area1 + area2 + area3

            GMTR_t(ij,k0,l,t,GMTR_t_AREA)  = area
            GMTR_t(ij,k0,l,t,GMTR_t_RAREA) = 1.0_RP / area

            GMTR_t(ij,k0,l,t,GMTR_t_W1)    = area1 / area
            GMTR_t(ij,k0,l,t,GMTR_t_W2)    = area2 / area
            GMTR_t(ij,k0,l,t,GMTR_t_W3)    = area3 / area
         enddo
         enddo
         enddo
      else
         do t = TI,TJ
         do j = ADM_gmin-1, ADM_gmax
         do i = ADM_gmin-1, ADM_gmax
            ij = suf(i,j)

            wk(:,:,ij,t) = wk(:,:,ij,t) / GRD_rscale

            area1 = VECTR_triangle( wk(:,0,ij,t), wk(:,2,ij,t), wk(:,3,ij,t), GMTR_polygon_type, GRD_rscale )
            area2 = VECTR_triangle( wk(:,0,ij,t), wk(:,3,ij,t), wk(:,1,ij,t), GMTR_polygon_type, GRD_rscale )
            area3 = VECTR_triangle( wk(:,0,ij,t), wk(:,1,ij,t), wk(:,2,ij,t), GMTR_polygon_type, GRD_rscale )

            area = area1 + area2 + area3

            GMTR_t(ij,k0,l,t,GMTR_t_AREA)  = area
            GMTR_t(ij,k0,l,t,GMTR_t_RAREA) = 1.0_RP / area

            GMTR_t(ij,k0,l,t,GMTR_t_W1)    = area1 / area
            GMTR_t(ij,k0,l,t,GMTR_t_W2)    = area2 / area
            GMTR_t(ij,k0,l,t,GMTR_t_W3)    = area3 / area
         enddo
         enddo
         enddo
      endif

   enddo

```

This section calculates an area of the triangle each triangle point represents. Note that `GRD_grid_type == GRD_grid_type_on_plane` is false in this kernel program. The triangle is divided to three small triangles by connecting three vertices and the triangle point. The area of whole triangle is calculated by summing up these three small triangles. `GMTR_t(:,:,:,:,GMTR_t_RAREA)` means the reciprocal of the area of whole triangle, and `GMTR_t(:,:,:,:,GMTR_t_W1)` etc. are the area fraction of each of three small triangles. These are used for the triangle linear interpolation from the grid point (the center of control cell) to the vertex point.

The remaining part of this subroutine is for the pole region and is as follows.

```
  if ( ADM_have_pl ) then
     n = ADM_gslf_pl

     do l = 1,ADM_lall_pl
        do v = ADM_gmin_pl, ADM_gmax_pl
           ij   = v
           ijp1 = v + 1
           if( ijp1 == ADM_gmax_pl+1 ) ijp1 = ADM_gmin_pl

           do d = 1, ADM_nxyz
              wk_pl(d,0) = GRD_xt_pl(ij,k0,l,d)

              wk_pl(d,1) = GRD_x_pl(n   ,k0,l,d)
              wk_pl(d,2) = GRD_x_pl(ij  ,k0,l,d)
              wk_pl(d,3) = GRD_x_pl(ijp1,k0,l,d)
           enddo

           wk_pl(:,:) = wk_pl(:,:) / GRD_rscale

           area1 = VECTR_triangle( wk_pl(:,0), wk_pl(:,2), wk_pl(:,3), GMTR_polygon_type, GRD_rscale )
           area2 = VECTR_triangle( wk_pl(:,0), wk_pl(:,3), wk_pl(:,1), GMTR_polygon_type, GRD_rscale )
           area3 = VECTR_triangle( wk_pl(:,0), wk_pl(:,1), wk_pl(:,2), GMTR_polygon_type, GRD_rscale )

           area = area1 + area2 + area3

           GMTR_t_pl(ij,k0,l,GMTR_t_AREA)  = area
           GMTR_t_pl(ij,k0,l,GMTR_t_RAREA) = 1.0_RP / area

           GMTR_t_pl(ij,k0,l,GMTR_t_W1)    = area1 / area
           GMTR_t_pl(ij,k0,l,GMTR_t_W2)    = area2 / area
           GMTR_t_pl(ij,k0,l,GMTR_t_W3)    = area3 / area
        enddo
     enddo
  endif

  return
end subroutine GMTR_t_setup
```

The procedure of calculation is almost the same with for the normal region, but note that for the pole region, there is only one triangle point for one grid point, and there is no distinction between `TI` and `TJ`.

(3) `GMTR_a_setup`

This subroutine is to setup metrics for the cell edge points or the arcs of triangles.

Argument lists and local variables definition part of this subroutine is as follows.

```
subroutine GMTR_a_setup( &
     GRD_x,   GRD_x_pl,   &
     GRD_xt, GRD_xt_pl, &
     GMTR_a, GMTR_a_pl, &
     GRD_rscale         )
!ESC!    use mod_adm, only: &
!ESC!       ADM_nxyz,      &
!ESC!       ADM_have_pl,   &
!ESC!       ADM_have_sgp, &
!ESC!       ADM_gmin,      &
!ESC!       ADM_gmax,      &
!ESC!       ADM_gslf_pl,   &
!ESC!       ADM_gmin_pl,   &
!ESC!       ADM_gmax_pl
!ESC!    use mod_grd, only: &
!ESC!       GRD_grid_type_on_plane, &
!ESC!       GRD_grid_type
  implicit none

  real(RP), intent(in)  :: GRD_x    (ADM_gall   ,k0,ADM_lall   ,              ADM_nxyz)
```

```
  real(RP), intent(in)  :: GRD_x_pl (ADM_gall_pl,k0,ADM_lall_pl,             ADM_nxyz)
  real(RP), intent(in)  :: GRD_xt   (ADM_gall   ,k0,ADM_lall   ,TI:TJ,ADM_nxyz)
  real(RP), intent(in)  :: GRD_xt_pl(ADM_gall_pl,k0,ADM_lall_pl,             ADM_nxyz)
  real(RP), intent(out) :: GMTR_a   (ADM_gall   ,k0,ADM_lall   ,AI:AJ,GMTR_a_nmax   )
  real(RP), intent(out) :: GMTR_a_pl(ADM_gall_pl,k0,ADM_lall_pl,             GMTR_a_nmax_pl)
  real(RP), intent(in)  :: GRD_rscale

  real(RP) :: wk   (ADM_nxyz,2,ADM_gall)
  real(RP) :: wk_pl(ADM_nxyz,2)

  real(RP) :: Tvec(3), Nvec(3)

  integer  :: ij
  integer  :: ip1j, ijp1, ip1jp1
  integer  :: im1j, ijm1

  integer  :: i, j, l, d, v, n
  !---------------------------------------------------------------------------

```

Input arguments are the same with the previous `GMTR_p_setup`. `GMTR_a` and `GMTR_a_pl` are the metrics for the edge of the triangle or the hexagonal/pentagonal control cell for the normal region and the pole region, respectively. In the last dimension of these arrays whose size is `GMTR_a_nmax`, each element specifies the kind of various metrics.

Main part of this subroutine is consist of two l-loops. The first loop is divided by three sections, and is as follows.

```
  !if( IO_L ) write(IO_FID_LOG,*) '*** setup metrics for cell arcs'

  GMTR_a   (:,:,:,:,:) = 0.0_RP
  GMTR_a_pl(:,:,:,:)   = 0.0_RP

  !--- Triangle
  do l = 1, ADM_lall

     !--- AI
     do j = ADM_gmin-1, ADM_gmax+1
     do i = ADM_gmin-1, ADM_gmax
        ij   = suf(i  ,j  )
        ip1j = suf(i+1,j  )

        do d = 1, ADM_nxyz
           wk(d,1,ij) = GRD_x(ij  ,k0,l,d)
           wk(d,2,ij) = GRD_x(ip1j,k0,l,d)
        enddo
     enddo
     enddo

     ! treat arc of unused triangle
     wk(:,1,suf(ADM_gmax  ,ADM_gmin-1)) = GRD_x(suf(ADM_gmax  ,ADM_gmin-1),k0,l,:)
     wk(:,2,suf(ADM_gmax  ,ADM_gmin-1)) = GRD_x(suf(ADM_gmax  ,ADM_gmin  ),k0,l,:)
     wk(:,1,suf(ADM_gmin-1,ADM_gmax+1)) = GRD_x(suf(ADM_gmin  ,ADM_gmax+1),k0,l,:)
     wk(:,2,suf(ADM_gmin-1,ADM_gmax+1)) = GRD_x(suf(ADM_gmin  ,ADM_gmax  ),k0,l,:)

     if ( ADM_have_sgp(l) ) then ! pentagon
        wk(:,1,suf(ADM_gmin-1,ADM_gmin-1)) = GRD_x(suf(ADM_gmin  ,ADM_gmin-1),k0,l,:)
        wk(:,2,suf(ADM_gmin-1,ADM_gmin-1)) = GRD_x(suf(ADM_gmin+1,ADM_gmin  ),k0,l,:)
     endif

     do j = ADM_gmin-1, ADM_gmax+1
     do i = ADM_gmin-1, ADM_gmax
        ij = suf(i,j)

        call GMTR_TNvec( Tvec(:), Nvec(:),                          & ! [OUT]
                         wk(:,1,ij), wk(:,2,ij),                    & ! [IN]
                         GRD_grid_type, GMTR_polygon_type, GRD_rscale ) ! [IN]

        GMTR_a(ij,k0,l,AI,GMTR_a_TNX) = Nvec(1)
        GMTR_a(ij,k0,l,AI,GMTR_a_TNY) = Nvec(2)
        GMTR_a(ij,k0,l,AI,GMTR_a_TNZ) = Nvec(3)
        GMTR_a(ij,k0,l,AI,GMTR_a_TTX) = Tvec(1)
        GMTR_a(ij,k0,l,AI,GMTR_a_TTY) = Tvec(2)
        GMTR_a(ij,k0,l,AI,GMTR_a_TTZ) = Tvec(3)
     enddo
     enddo

     !--- AIJ
```

```
      do j = ADM_gmin-1, ADM_gmax
      do i = ADM_gmin-1, ADM_gmax
         ij     = suf(i  ,j  )
         ip1jp1 = suf(i+1,j+1)

         do d = 1, ADM_nxyz
            wk(d,1,ij) = GRD_x(ij    ,k0,l,d)
            wk(d,2,ij) = GRD_x(ip1jp1,k0,l,d)
         enddo
      enddo
      enddo

      do j = ADM_gmin-1, ADM_gmax
      do i = ADM_gmin-1, ADM_gmax
         ij = suf(i,j)

         call GMTR_TNvec( Tvec(:), Nvec(:),                            & ! [OUT]
                          wk(:,1,ij), wk(:,2,ij),                      & ! [IN]
                          GRD_grid_type, GMTR_polygon_type, GRD_rscale ) ! [IN]

         GMTR_a(ij,k0,l,AIJ,GMTR_a_TNX) = Nvec(1)
         GMTR_a(ij,k0,l,AIJ,GMTR_a_TNY) = Nvec(2)
         GMTR_a(ij,k0,l,AIJ,GMTR_a_TNZ) = Nvec(3)
         GMTR_a(ij,k0,l,AIJ,GMTR_a_TTX) = Tvec(1)
         GMTR_a(ij,k0,l,AIJ,GMTR_a_TTY) = Tvec(2)
         GMTR_a(ij,k0,l,AIJ,GMTR_a_TTZ) = Tvec(3)
      enddo
      enddo

      !--- AJ
      do j = ADM_gmin-1, ADM_gmax
      do i = ADM_gmin-1, ADM_gmax+1
         ij   = suf(i  ,j  )
         ijp1 = suf(i  ,j+1)

         do d = 1, ADM_nxyz
            wk(d,1,ij) = GRD_x(ij  ,k0,l,d)
            wk(d,2,ij) = GRD_x(ijp1,k0,l,d)
         enddo
      enddo
      enddo

      ! treat arc of unused triangle
      wk(:,1,suf(ADM_gmax+1,ADM_gmin-1)) = GRD_x(suf(ADM_gmax+1,ADM_gmin),k0,l,:)
      wk(:,2,suf(ADM_gmax+1,ADM_gmin-1)) = GRD_x(suf(ADM_gmax  ,ADM_gmin),k0,l,:)
      wk(:,1,suf(ADM_gmin-1,ADM_gmax  )) = GRD_x(suf(ADM_gmin-1,ADM_gmax),k0,l,:)
      wk(:,2,suf(ADM_gmin-1,ADM_gmax  )) = GRD_x(suf(ADM_gmin  ,ADM_gmax),k0,l,:)

      do j = ADM_gmin-1, ADM_gmax
      do i = ADM_gmin-1, ADM_gmax+1
         ij = suf(i,j)

         call GMTR_TNvec( Tvec(:), Nvec(:),                            & ! [OUT]
                          wk(:,1,ij), wk(:,2,ij),                      & ! [IN]
                          GRD_grid_type, GMTR_polygon_type, GRD_rscale ) ! [IN]

         GMTR_a(ij,k0,l,AJ,GMTR_a_TNX) = Nvec(1)
         GMTR_a(ij,k0,l,AJ,GMTR_a_TNY) = Nvec(2)
         GMTR_a(ij,k0,l,AJ,GMTR_a_TNZ) = Nvec(3)
         GMTR_a(ij,k0,l,AJ,GMTR_a_TTX) = Tvec(1)
         GMTR_a(ij,k0,l,AJ,GMTR_a_TTY) = Tvec(2)
         GMTR_a(ij,k0,l,AJ,GMTR_a_TTZ) = Tvec(3)
      enddo
      enddo

   enddo ! l loop
```

These three sections calculate the metrics for the edge points of three arcs of triangle represented by `AI`, `AIJ` and `AJ`, respectively. After setting the coordinates of two endpoints of the arc (`wk`), subroutine `GMTR_TNvec` calculates a normal vector and a tangent vector of the edge.

The second loop is alose divided by three sections, and is as follows.

```
   !--- Hexagon/Pentagon
   do l = 1, ADM_lall

      !--- AI
      do j = ADM_gmin,   ADM_gmax
      do i = ADM_gmin-1, ADM_gmax
```

```
         ij   = suf(i  ,j  )
         ijm1 = suf(i  ,j-1)

         do d = 1, ADM_nxyz
            wk(d,1,ij) = GRD_xt(ij  ,k0,l,TI,d)
            wk(d,2,ij) = GRD_xt(ijm1,k0,l,TJ,d)
         enddo
      enddo
      enddo

      do j = ADM_gmin,   ADM_gmax
      do i = ADM_gmin-1, ADM_gmax
         ij = suf(i,j)

         call GMTR_TNvec( Tvec(:), Nvec(:),                           & ! [OUT]
                          wk(:,1,ij), wk(:,2,ij),                     & ! [IN]
                          GRD_grid_type, GMTR_polygon_type, GRD_rscale ) ! [IN]

         GMTR_a(ij,k0,l,AI,GMTR_a_HNX) = Nvec(1)
         GMTR_a(ij,k0,l,AI,GMTR_a_HNY) = Nvec(2)
         GMTR_a(ij,k0,l,AI,GMTR_a_HNZ) = Nvec(3)
         GMTR_a(ij,k0,l,AI,GMTR_a_HTX) = Tvec(1)
         GMTR_a(ij,k0,l,AI,GMTR_a_HTY) = Tvec(2)
         GMTR_a(ij,k0,l,AI,GMTR_a_HTZ) = Tvec(3)
      enddo
      enddo

      !--- AIJ
      do j = ADM_gmin-1, ADM_gmax
      do i = ADM_gmin-1, ADM_gmax
         ij   = suf(i  ,j  )

         do d = 1, ADM_nxyz
            wk(d,1,ij) = GRD_xt(ij  ,k0,l,TJ,d)
            wk(d,2,ij) = GRD_xt(ij  ,k0,l,TI,d)
         enddo
      enddo
      enddo

      ! treat arc of unused hexagon
      wk(:,1,suf(ADM_gmax  ,ADM_gmin-1)) = GRD_xt(suf(ADM_gmax  ,ADM_gmin-1),k0,l,TJ,:)
      wk(:,2,suf(ADM_gmax  ,ADM_gmin-1)) = GRD_xt(suf(ADM_gmax  ,ADM_gmin  ),k0,l,TI,:)
      wk(:,1,suf(ADM_gmin-1,ADM_gmax  )) = GRD_xt(suf(ADM_gmin  ,ADM_gmax  ),k0,l,TJ,:)
      wk(:,2,suf(ADM_gmin-1,ADM_gmax  )) = GRD_xt(suf(ADM_gmin-1,ADM_gmax  ),k0,l,TI,:)

      do j = ADM_gmin-1, ADM_gmax
      do i = ADM_gmin-1, ADM_gmax
         ij = suf(i,j)

         call GMTR_TNvec( Tvec(:), Nvec(:),                           & ! [OUT]
                          wk(:,1,ij), wk(:,2,ij),                     & ! [IN]
                          GRD_grid_type, GMTR_polygon_type, GRD_rscale ) ! [IN]

         GMTR_a(ij,k0,l,AIJ,GMTR_a_HNX) = Nvec(1)
         GMTR_a(ij,k0,l,AIJ,GMTR_a_HNY) = Nvec(2)
         GMTR_a(ij,k0,l,AIJ,GMTR_a_HNZ) = Nvec(3)
         GMTR_a(ij,k0,l,AIJ,GMTR_a_HTX) = Tvec(1)
         GMTR_a(ij,k0,l,AIJ,GMTR_a_HTY) = Tvec(2)
         GMTR_a(ij,k0,l,AIJ,GMTR_a_HTZ) = Tvec(3)
      enddo
      enddo

      !--- AJ
      do j = ADM_gmin-1, ADM_gmax
      do i = ADM_gmin,   ADM_gmax
         ij   = suf(i  ,j  )
         im1j = suf(i-1,j  )

         do d = 1, ADM_nxyz
            wk(d,1,ij) = GRD_xt(im1j,k0,l,TI,d)
            wk(d,2,ij) = GRD_xt(ij  ,k0,l,TJ,d)
         enddo
      enddo
      enddo

      if ( ADM_have_sgp(l) ) then ! pentagon
         wk(:,1,suf(ADM_gmin  ,ADM_gmin-1)) = GRD_xt(suf(ADM_gmin  ,ADM_gmin  ),k0,l,TI,:)
         wk(:,2,suf(ADM_gmin  ,ADM_gmin-1)) = GRD_xt(suf(ADM_gmin  ,ADM_gmin-1),k0,l,TJ,:)
      endif

      do j = ADM_gmin-1, ADM_gmax
      do i = ADM_gmin,   ADM_gmax
```

```
      ij = suf(i,j)

      call GMTR_TNvec( Tvec(:), Nvec(:),                          & ! [OUT]
                       wk(:,1,ij), wk(:,2,ij),                    & ! [IN]
                       GRD_grid_type, GMTR_polygon_type, GRD_rscale ) ! [IN]

      GMTR_a(ij,k0,l,AJ,GMTR_a_HNX) = Nvec(1)
      GMTR_a(ij,k0,l,AJ,GMTR_a_HNY) = Nvec(2)
      GMTR_a(ij,k0,l,AJ,GMTR_a_HNZ) = Nvec(3)
      GMTR_a(ij,k0,l,AJ,GMTR_a_HTX) = Tvec(1)
      GMTR_a(ij,k0,l,AJ,GMTR_a_HTY) = Tvec(2)
      GMTR_a(ij,k0,l,AJ,GMTR_a_HTZ) = Tvec(3)
    enddo
    enddo

  enddo ! l loop

```

These three sections also calculate the metrics for the three arc points represented by `AI`, `AIJ` and `AJ`, respectively, but these sections are for the edges of hexagonal/pentagonal control cell, i.e. the line crossing the arc of triangle. After setting the coordinates of two vertex points, subroutine `GMTR_TNvec` calculates a normal vector and a tangent vector of the line connecting two endpoints.

The remaining part of this subroutine is for the pole region, and is as follows.

```
  if ( ADM_have_pl ) then
    n = ADM_gslf_pl

    do l = 1, ADM_lall_pl

      !--- Triangle (arc 1)
      do v = ADM_gmin_pl, ADM_gmax_pl
        ij = v

        do d = 1, ADM_nxyz
          wk_pl(d,1) = GRD_x_pl(n ,k0,l,d)
          wk_pl(d,2) = GRD_x_pl(ij,k0,l,d)
        enddo

        call GMTR_TNvec( Tvec(:), Nvec(:),                          & ! [OUT]
                         wk_pl(:,1), wk_pl(:,2),                    & ! [IN]
                         GRD_grid_type, GMTR_polygon_type, GRD_rscale ) ! [IN]

        GMTR_a_pl(ij,k0,l,GMTR_a_TNX) = Nvec(1)
        GMTR_a_pl(ij,k0,l,GMTR_a_TNY) = Nvec(2)
        GMTR_a_pl(ij,k0,l,GMTR_a_TNZ) = Nvec(3)
        GMTR_a_pl(ij,k0,l,GMTR_a_TTX) = Tvec(1)
        GMTR_a_pl(ij,k0,l,GMTR_a_TTY) = Tvec(2)
        GMTR_a_pl(ij,k0,l,GMTR_a_TTZ) = Tvec(3)
      enddo

      !--- Triangle (arc 2)
      do v = ADM_gmin_pl, ADM_gmax_pl
        ij   = v
        ijp1 = v+1
        if ( ijp1 == ADM_gmax_pl+1 ) ijp1 = ADM_gmin_pl

        do d = 1, ADM_nxyz
          wk_pl(d,1) = GRD_x_pl(ij  ,k0,l,d)
          wk_pl(d,2) = GRD_x_pl(ijp1,k0,l,d)
        enddo

        call GMTR_TNvec( Tvec(:), Nvec(:),                          & ! [OUT]
                         wk_pl(:,1), wk_pl(:,2),                    & ! [IN]
                         GRD_grid_type, GMTR_polygon_type, GRD_rscale ) ! [IN]

        GMTR_a_pl(ij,k0,l,GMTR_a_TN2X) = Nvec(1)
        GMTR_a_pl(ij,k0,l,GMTR_a_TN2Y) = Nvec(2)
        GMTR_a_pl(ij,k0,l,GMTR_a_TN2Z) = Nvec(3)
        GMTR_a_pl(ij,k0,l,GMTR_a_TT2X) = Tvec(1)
        GMTR_a_pl(ij,k0,l,GMTR_a_TT2Y) = Tvec(2)
        GMTR_a_pl(ij,k0,l,GMTR_a_TT2Z) = Tvec(3)
      enddo

      !--- Pentagon
      do v = ADM_gmin_pl, ADM_gmax_pl
        ij   = v
        ijm1 = v-1
        if ( ijm1 == ADM_gmin_pl-1 ) ijm1 = ADM_gmax_pl
```

```
            do d = 1, ADM_nxyz
               wk_pl(d,1) = GRD_xt_pl(ijm1,k0,l,d)
               wk_pl(d,2) = GRD_xt_pl(ij  ,k0,l,d)
            enddo

            call GMTR_TNvec( Tvec(:), Nvec(:),                           & ! [OUT]
                             wk_pl(:,1), wk_pl(:,2),                     & ! [IN]
                             GRD_grid_type, GMTR_polygon_type, GRD_rscale ) ! [IN]

            GMTR_a_pl(ij,k0,l,GMTR_a_HNX) = Nvec(1)
            GMTR_a_pl(ij,k0,l,GMTR_a_HNY) = Nvec(2)
            GMTR_a_pl(ij,k0,l,GMTR_a_HNZ) = Nvec(3)
            GMTR_a_pl(ij,k0,l,GMTR_a_HTX) = Tvec(1)
            GMTR_a_pl(ij,k0,l,GMTR_a_HTY) = Tvec(2)
            GMTR_a_pl(ij,k0,l,GMTR_a_HTZ) = Tvec(3)
         enddo

      enddo
   endif

   return
end subroutine GMTR_a_setup
```

The normal vector and the tangent vector are calculated between the pole point and neighboring grid point, and between the two neighboring grid points. Then the metrics for the edge of the pentagonal control cell are calculated by almost the same procedure using subroutine `GMTR_TNvec`.

(4) `OPRT_divergence_setup`

Module `mod_oprt` contains public objects related to the vector operator, such as divergence, rotation, laplacian, etc. Calculation of these vector operator is descretized to form stencil calculation, which needs stencil coefficients. This subroutine calculates the coefficients for the divergence operator, subroutine `OPRT_divergence`, in advance.

Argument lists and local variables definition part of this subroutine is as follows.

```
subroutine OPRT_divergence_setup( &
     GMTR_p,   GMTR_p_pl,  &
     GMTR_t,   GMTR_t_pl,  &
     GMTR_a,   GMTR_a_pl,  &
     coef_div, coef_div_pl )
!ESC!    use mod_adm, only: &
!ESC!       ADM_have_pl,  &
!ESC!       ADM_have_sgp, &
!ESC!       ADM_gall_1d,  &
!ESC!       ADM_gmin,     &
!ESC!       ADM_gmax,     &
!ESC!       ADM_gslf_pl,  &
!ESC!       ADM_gmin_pl,  &
!ESC!       ADM_gmax_pl
!ESC!    use mod_gmtr, only: &
!ESC!       P_RAREA => GMTR_p_RAREA, &
!ESC!       W1      => GMTR_t_W1,    &
!ESC!       W2      => GMTR_t_W2,    &
!ESC!       W3      => GMTR_t_W3,    &
!ESC!       HNX     => GMTR_a_HNX,   &
!ESC!       GMTR_p_nmax,             &
!ESC!       GMTR_t_nmax,             &
!ESC!       GMTR_a_nmax,             &
!ESC!       GMTR_a_nmax_pl
  implicit none

  real(RP), intent(in)  :: GMTR_p     (ADM_gall   ,K0,ADM_lall   ,      GMTR_p_nmax   )
  real(RP), intent(in)  :: GMTR_p_pl  (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_p_nmax   )
  real(RP), intent(in)  :: GMTR_t     (ADM_gall   ,K0,ADM_lall   ,TI:TJ,GMTR_t_nmax   )
  real(RP), intent(in)  :: GMTR_t_pl  (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_t_nmax   )
  real(RP), intent(in)  :: GMTR_a     (ADM_gall   ,K0,ADM_lall   ,AI:AJ,GMTR_a_nmax   )
  real(RP), intent(in)  :: GMTR_a_pl  (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_a_nmax_pl)
  real(RP), intent(out) :: coef_div   (ADM_gall,0:6        ,ADM_nxyz,ADM_lall   )
  real(RP), intent(out) :: coef_div_pl(         0:ADM_vlink,ADM_nxyz,ADM_lall_pl)

  integer  :: gmin, gmax, iall, gall, nxyz, lall

  integer  :: ij
  integer  :: ip1j, ijp1, ip1jp1
```

```
    integer  :: im1j, ijm1, im1jm1

    real(RP) :: coef
    integer  :: g, l, d, n, v, hn
    !---------------------------------------------------------------------------

```

Input arguments are the metrics calculated by the subroutine described before in this section. Output arguments `coef_div` and `coef_div_pl` are the coefficients for the divergence operator for the normal region and the pole region, respectively. The second dimension of `coef_div` has the range `0:6`, which corresponds to the 7-point stencil calculation. Also the second dimension of `coef_div_pl` has the range `0:ADM_vlink` where `ADM_vlink` is 5, corresponds to the 6-point stencil calculation. Note that the pole region is a pentagon.

The procedure of this subroutine is consist of three parts. The first part is as follows.

```
    !if( IO_L ) write(IO_FID_LOG,*) '*** setup coefficient of divergence operator'

    gmin = (ADM_gmin-1)*ADM_gall_1d + ADM_gmin
    gmax = (ADM_gmax-1)*ADM_gall_1d + ADM_gmax
    iall = ADM_gall_1d
    gall = ADM_gall
    nxyz = ADM_nxyz
    lall = ADM_lall

    !$omp parallel workshare
    coef_div   (:,:,:,:) = 0.0_RP
    !$omp end parallel workshare
    coef_div_pl(  :,:,:) = 0.0_RP

    !$omp parallel default(none),private(g,d,l,hn,ij,ip1j,ip1jp1,ijp1,im1j,ijm1,im1jm1), &
    !$omp shared(ADM_have_sgp,gmin,gmax,iall,gall,nxyz,lall,coef_div,GMTR_p,GMTR_t,GMTR_a)
    do l = 1, lall
    do d = 1, nxyz
       hn = d + HNX - 1

       !$omp do
       do g = gmin, gmax
          ij     = g
          ip1j   = g + 1
          ip1jp1 = g + iall + 1
          ijp1   = g + iall
          im1j   = g - 1
          im1jm1 = g - iall - 1
          ijm1   = g - iall

          ! ij
          coef_div(ij,0,d,l) = ( + GMTR_t(ij    ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q1 * b6
                                 + GMTR_t(ij    ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q1 * b1
                                 + GMTR_t(ij    ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q2 * b1
                                 + GMTR_t(ij    ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q2 * b2
                                 + GMTR_t(im1j  ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q3 * b2
                                 - GMTR_t(im1j  ,k0,l,TI,W2) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q3 * b3
                                 - GMTR_t(im1jm1,k0,l,TJ,W2) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q4 * b3
                                 - GMTR_t(im1jm1,k0,l,TJ,W2) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q4 * b4
                                 - GMTR_t(im1jm1,k0,l,TI,W3) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q5 * b4
                                 - GMTR_t(im1jm1,k0,l,TI,W3) * GMTR_a(ijm1  ,k0,l,AJ ,hn) & ! Q5 * b5
                                 - GMTR_t(ijm1  ,k0,l,TJ,W3) * GMTR_a(ijm1  ,k0,l,AJ ,hn) & ! Q6 * b5
                                 + GMTR_t(ijm1  ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q6 * b6
                               ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
          ! ip1j
          coef_div(ij,1,d,l) = ( - GMTR_t(ijm1  ,k0,l,TJ,W2) * GMTR_a(ijm1  ,k0,l,AJ ,hn) & ! Q6 * b5
                                 + GMTR_t(ijm1  ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q6 * b6
                                 + GMTR_t(ij    ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q1 * b6
                                 + GMTR_t(ij    ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q1 * b1
                               ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
          ! ip1jp1
          coef_div(ij,2,d,l) = ( + GMTR_t(ij    ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q1 * b6
                                 + GMTR_t(ij    ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q1 * b1
                                 + GMTR_t(ij    ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q2 * b1
                                 + GMTR_t(ij    ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q2 * b2
                               ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
          ! ijp1
          coef_div(ij,3,d,l) = ( + GMTR_t(ij    ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q2 * b1
                                 + GMTR_t(ij    ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q2 * b2
                                 + GMTR_t(im1j  ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q3 * b2
                                 - GMTR_t(im1j  ,k0,l,TI,W3) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q3 * b3
                               ) * 0.5_RP*GMTR_p(ij,k0,l,P_RAREA)
          ! im1j
```

```
      coef_div(ij,4,d,l) = ( + GMTR_t(im1j  ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q3 * b2
                             - GMTR_t(im1j  ,k0,l,TI,W1) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q3 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W3) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q4 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W3) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q4 * b4
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! im1jm1
      coef_div(ij,5,d,l) = ( - GMTR_t(im1jm1,k0,l,TJ,W1) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q4 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W1) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q4 * b4
                             - GMTR_t(im1jm1,k0,l,TI,W1) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q5 * b4
                             - GMTR_t(im1jm1,k0,l,TI,W1) * GMTR_a(ijm1  ,k0,l,AJ ,hn) & ! Q5 * b5
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! ijm1
      coef_div(ij,6,d,l) = ( - GMTR_t(im1jm1,k0,l,TI,W2) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q5 * b4
                             - GMTR_t(im1jm1,k0,l,TI,W2) * GMTR_a(ijm1  ,k0,l,AJ ,hn) & ! Q5 * b5
                             - GMTR_t(ijm1  ,k0,l,TJ,W1) * GMTR_a(ijm1  ,k0,l,AJ ,hn) & ! Q6 * b5
                             + GMTR_t(ijm1  ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q6 * b6
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
    enddo
    !$omp end do

```

The first part above and the second part below are in a long l-loop. In the g-loop represents horizontal index, 7 coefficients are calculated separately from the various metrics.

The second part is as follows.

```
    if ( ADM_have_sgp(l) ) then ! pentagon
      !$omp master
      ij     = gmin
      ip1j   = gmin + 1
      ip1jp1 = gmin + iall + 1
      ijp1   = gmin + iall
      im1j   = gmin - 1
      im1jm1 = gmin - iall - 1
      ijm1   = gmin - iall

      ! ij
      coef_div(ij,0,d,l) = ( + GMTR_t(ij    ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q1 * b6
                             + GMTR_t(ij    ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q1 * b1
                             + GMTR_t(ij    ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q2 * b1
                             + GMTR_t(ij    ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q2 * b2
                             + GMTR_t(im1j  ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q3 * b2
                             - GMTR_t(im1j  ,k0,l,TI,W2) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q3 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W2) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q4 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W2) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q4 * b4
                             - GMTR_t(ijm1  ,k0,l,TJ,W3) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q6 * b4
                             + GMTR_t(ijm1  ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q6 * b6
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! ip1j
      coef_div(ij,1,d,l) = ( - GMTR_t(ijm1  ,k0,l,TJ,W2) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q6 * b4
                             + GMTR_t(ijm1  ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q6 * b6
                             + GMTR_t(ij    ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q1 * b6
                             + GMTR_t(ij    ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q1 * b1
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! ip1jp1
      coef_div(ij,2,d,l) = ( + GMTR_t(ij    ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q1 * b6
                             + GMTR_t(ij    ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q1 * b1
                             + GMTR_t(ij    ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q2 * b1
                             + GMTR_t(ij    ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q2 * b2
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! ijp1
      coef_div(ij,3,d,l) = ( + GMTR_t(ij    ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q2 * b1
                             + GMTR_t(ij    ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q2 * b2
                             + GMTR_t(im1j  ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q3 * b2
                             - GMTR_t(im1j  ,k0,l,TI,W3) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q3 * b3
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! im1j
      coef_div(ij,4,d,l) = ( + GMTR_t(im1j  ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q3 * b2
                             - GMTR_t(im1j  ,k0,l,TI,W1) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q3 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W3) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q4 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W3) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q4 * b4
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! im1jm1
      coef_div(ij,5,d,l) = ( - GMTR_t(im1jm1,k0,l,TJ,W1) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q4 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W1) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q4 * b4
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! ijm1
      coef_div(ij,6,d,l) = ( - GMTR_t(ijm1  ,k0,l,TJ,W1) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q6 * b4
                             + GMTR_t(ijm1  ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q6 * b6
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
```

```
      !$omp end master
   endif

enddo ! loop d
enddo ! loop l
!$omp end parallel
```

This section is for the singular point. Note that `ADM_have_sgp(1)` is true.

The last section of this subroutine is for the pole region and is as follows.

```
   if ( ADM_have_pl ) then
      n = ADM_gslf_pl
      do l = 1, ADM_lall_pl
      do d = 1, ADM_nxyz
         hn = d + HNX - 1

         coef = 0.0_RP
         do v = ADM_gmin_pl, ADM_gmax_pl
            ij   = v
            ijp1 = v + 1
            if( ijp1 == ADM_gmax_pl+1 ) ijp1 = ADM_gmin_pl

            coef = coef + ( GMTR_t_pl(ij,k0,l,W1) * GMTR_a_pl(ij  ,k0,l,hn) &
                          + GMTR_t_pl(ij,k0,l,W1) * GMTR_a_pl(ijp1,k0,l,hn) )
         enddo
         coef_div_pl(0,d,l) = coef * 0.5_RP * GMTR_p_pl(n,k0,l,P_RAREA)

         do v = ADM_gmin_pl, ADM_gmax_pl
            ij   = v
            ijp1 = v + 1
            ijm1 = v - 1
            if( ijp1 == ADM_gmax_pl + 1 ) ijp1 = ADM_gmin_pl
            if( ijm1 == ADM_gmin_pl - 1 ) ijm1 = ADM_gmax_pl

            coef_div_pl(v-1,d,l) = ( + GMTR_t_pl(ijm1,k0,l,W3) * GMTR_a_pl(ijm1,k0,l,hn) &
                                     + GMTR_t_pl(ijm1,k0,l,W3) * GMTR_a_pl(ij  ,k0,l,hn) &
                                     + GMTR_t_pl(ij  ,k0,l,W2) * GMTR_a_pl(ij  ,k0,l,hn) &
                                     + GMTR_t_pl(ij  ,k0,l,W2) * GMTR_a_pl(ijp1,k0,l,hn) &
                                   ) * 0.5_RP * GMTR_p_pl(n,k0,l,P_RAREA)
         enddo
      enddo ! loop d
      enddo ! loop l
   endif

   return
end subroutine OPRT_divergence_setup
```

Note the data layout of coefficient is slightly different between the normal region and the pole region, First dimension of `coef_div_pl` corresponds to the second dimention of `coef_div`. The range of index `v` of the inner-most loop are `ADM_gmin_pl` and `ADM_gmax_pl`, which means the five grid points surrounding the pole point.

(5) `OPRT_rotation_setup`

This subroutine is similar to the previous one, but this one is for the rotation operator, subroutine `OPRT_rotation`.

Argument lists and local variables definition part of this subroutine is as follows.

```
subroutine OPRT_rotation_setup( &
     GMTR_p,   GMTR_p_pl,  &
     GMTR_t,   GMTR_t_pl,  &
     GMTR_a,   GMTR_a_pl,  &
     coef_rot, coef_rot_pl )
!ESC!    use mod_adm, only: &
!ESC!       ADM_have_pl,  &
!ESC!       ADM_have_sgp, &
!ESC!       ADM_gall_1d,  &
!ESC!       ADM_gmin,     &
!ESC!       ADM_gmax,     &
!ESC!       ADM_gslf_pl,  &
!ESC!       ADM_gmin_pl,  &
!ESC!       ADM_gmax_pl
!ESC!    use mod_gmtr, only: &
!ESC!       P_RAREA => GMTR_p_RAREA, &
```

```
!ESC!       W1      => GMTR_t_W1,     &
!ESC!       W2      => GMTR_t_W2,     &
!ESC!       W3      => GMTR_t_W3,     &
!ESC!       HTX     => GMTR_a_HTX,    &
!ESC!       GMTR_p_nmax,              &
!ESC!       GMTR_t_nmax,              &
!ESC!       GMTR_a_nmax,              &
!ESC!       GMTR_a_nmax_pl
  implicit none

  real(RP), intent(in)  :: GMTR_p     (ADM_gall   ,K0,ADM_lall   ,      GMTR_p_nmax   )
  real(RP), intent(in)  :: GMTR_p_pl  (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_p_nmax   )
  real(RP), intent(in)  :: GMTR_t     (ADM_gall   ,K0,ADM_lall   ,TI:TJ,GMTR_t_nmax   )
  real(RP), intent(in)  :: GMTR_t_pl  (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_t_nmax   )
  real(RP), intent(in)  :: GMTR_a     (ADM_gall   ,K0,ADM_lall   ,AI:AJ,GMTR_a_nmax   )
  real(RP), intent(in)  :: GMTR_a_pl  (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_a_nmax_pl)
  real(RP), intent(out) :: coef_rot   (ADM_gall,0:6        ,ADM_nxyz,ADM_lall   )
  real(RP), intent(out) :: coef_rot_pl(         0:ADM_vlink,ADM_nxyz,ADM_lall_pl)

  integer :: gmin, gmax, iall, gall, nxyz, lall

  integer  :: ij
  integer  :: ip1j, ijp1, ip1jp1
  integer  :: im1j, ijm1, im1jm1

  real(RP) :: coef
  integer  :: g, l, d, n, v, ht
  !---------------------------------------------------------------------------

```

Input arguments are the same with subroutine `OPRT_divergence_setup`, the metrics calculated by the subroutine described before in this section. Output arguments `coef_rot` and `coef_rot_pl` are the coefficients for the rotation operator for the normal region and the pole region, respectively. The second dimension of `coef_rot` has the range `0:6`, which corresponds to the 7-point stencil calculation. Also the second dimension of `coef_rot_pl` has the range `0:ADM_vlink` where `ADM_vlink` is 5, corresponds to the 6-point stencil calculation.

The procedure of this subroutine is consist of three parts. The first part is as follows.

```
  !if( IO_L ) write(IO_FID_LOG,*) '*** setup coefficient of rotation operator'

  gmin = (ADM_gmin-1)*ADM_gall_1d + ADM_gmin
  gmax = (ADM_gmax-1)*ADM_gall_1d + ADM_gmax
  iall = ADM_gall_1d
  gall = ADM_gall
  nxyz = ADM_nxyz
  lall = ADM_lall

  !$omp parallel workshare
  coef_rot   (:,:,:,:) = 0.0_RP
  !$omp end parallel workshare
  coef_rot_pl(  :,:,:) = 0.0_RP

  !$omp parallel default(none),private(g,d,l,ht,ij,ip1j,ip1jp1,ijp1,im1j,ijm1,im1jm1), &
  !$omp shared(ADM_have_sgp,gmin,gmax,iall,gall,nxyz,lall,coef_rot,GMTR_p,GMTR_t,GMTR_a)
  do l = 1, lall
  do d = 1, nxyz
     ht = d + HTX - 1

     !$omp do
     do g = gmin, gmax
        ij     = g
        ip1j   = g + 1
        ip1jp1 = g + iall + 1
        ijp1   = g + iall
        im1j   = g - 1
        im1jm1 = g - iall - 1
        ijm1   = g - iall

        ! ij
        coef_rot(ij,0,d,l) = ( + GMTR_t(ij    ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AI ,ht) & ! Q1 * b6
                               + GMTR_t(ij    ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AIJ,ht) & ! Q1 * b1
                               + GMTR_t(ij    ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AIJ,ht) & ! Q2 * b1
                               + GMTR_t(ij    ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AJ ,ht) & ! Q2 * b2
                               + GMTR_t(im1j  ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AJ ,ht) & ! Q3 * b2
                               - GMTR_t(im1j  ,k0,l,TI,W2) * GMTR_a(im1j  ,k0,l,AI ,ht) & ! Q3 * b3
                               - GMTR_t(im1jm1,k0,l,TJ,W2) * GMTR_a(im1j  ,k0,l,AI ,ht) & ! Q4 * b3
                               - GMTR_t(im1jm1,k0,l,TJ,W2) * GMTR_a(im1jm1,k0,l,AIJ,ht) & ! Q4 * b4
```

```
                             - GMTR_t(im1jm1,k0,l,TI,W3) * GMTR_a(im1jm1,k0,l,AIJ,ht) & ! Q5 * b4
                             - GMTR_t(im1jm1,k0,l,TI,W3) * GMTR_a(ijm1  ,k0,l,AJ ,ht) & ! Q5 * b5
                             - GMTR_t(ijm1  ,k0,l,TJ,W3) * GMTR_a(ijm1  ,k0,l,AJ ,ht) & ! Q6 * b5
                             + GMTR_t(ijm1  ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AI ,ht) & ! Q6 * b6
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! ip1j
      coef_rot(ij,1,d,l) = ( - GMTR_t(ijm1  ,k0,l,TJ,W2) * GMTR_a(ijm1  ,k0,l,AJ ,ht) & ! Q6 * b5
                             + GMTR_t(ijm1  ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AI ,ht) & ! Q6 * b6
                             + GMTR_t(ij    ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AI ,ht) & ! Q1 * b6
                             + GMTR_t(ij    ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AIJ,ht) & ! Q1 * b1
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! ip1jp1
      coef_rot(ij,2,d,l) = ( + GMTR_t(ij    ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AI ,ht) & ! Q1 * b6
                             + GMTR_t(ij    ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AIJ,ht) & ! Q1 * b1
                             + GMTR_t(ij    ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AIJ,ht) & ! Q2 * b1
                             + GMTR_t(ij    ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AJ ,ht) & ! Q2 * b2
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! ijp1
      coef_rot(ij,3,d,l) = ( + GMTR_t(ij    ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AIJ,ht) & ! Q2 * b1
                             + GMTR_t(ij    ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AJ ,ht) & ! Q2 * b2
                             + GMTR_t(im1j  ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AJ ,ht) & ! Q3 * b2
                             - GMTR_t(im1j  ,k0,l,TI,W3) * GMTR_a(im1j  ,k0,l,AI ,ht) & ! Q3 * b3
                           ) * 0.5_RP*GMTR_p(ij,k0,l,P_RAREA)
      ! im1j
      coef_rot(ij,4,d,l) = ( + GMTR_t(im1j  ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AJ ,ht) & ! Q3 * b2
                             - GMTR_t(im1j  ,k0,l,TI,W1) * GMTR_a(im1j  ,k0,l,AI ,ht) & ! Q3 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W3) * GMTR_a(im1j  ,k0,l,AI ,ht) & ! Q4 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W3) * GMTR_a(im1jm1,k0,l,AIJ,ht) & ! Q4 * b4
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! im1jm1
      coef_rot(ij,5,d,l) = ( - GMTR_t(im1jm1,k0,l,TJ,W1) * GMTR_a(im1j  ,k0,l,AI ,ht) & ! Q4 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W1) * GMTR_a(im1jm1,k0,l,AIJ,ht) & ! Q4 * b4
                             - GMTR_t(im1jm1,k0,l,TI,W1) * GMTR_a(im1jm1,k0,l,AIJ,ht) & ! Q5 * b4
                             - GMTR_t(im1jm1,k0,l,TI,W1) * GMTR_a(ijm1  ,k0,l,AJ ,ht) & ! Q5 * b5
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! ijm1
      coef_rot(ij,6,d,l) = ( - GMTR_t(im1jm1,k0,l,TI,W2) * GMTR_a(im1jm1,k0,l,AIJ,ht) & ! Q5 * b4
                             - GMTR_t(im1jm1,k0,l,TI,W2) * GMTR_a(ijm1  ,k0,l,AJ ,ht) & ! Q5 * b5
                             - GMTR_t(ijm1  ,k0,l,TJ,W1) * GMTR_a(ijm1  ,k0,l,AJ ,ht) & ! Q6 * b5
                             + GMTR_t(ijm1  ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AI ,ht) & ! Q6 * b6
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
    enddo
    !$omp end do

```

The first part above and the second part below are in a long l-loop. In the g-loop represents horizontal index, 7 coefficients are calculated separately from the various metrics.

The second part is as follows.

```
    if ( ADM_have_sgp(l) ) then ! pentagon
      !$omp master
      ij     = gmin
      ip1j   = gmin + 1
      ip1jp1 = gmin + iall + 1
      ijp1   = gmin + iall
      im1j   = gmin - 1
      im1jm1 = gmin - iall - 1
      ijm1   = gmin - iall

      ! ij
      coef_rot(ij,0,d,l) = ( + GMTR_t(ij    ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AI ,ht) & ! Q1 * b6
                             + GMTR_t(ij    ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AIJ,ht) & ! Q1 * b1
                             + GMTR_t(ij    ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AIJ,ht) & ! Q2 * b1
                             + GMTR_t(ij    ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AJ ,ht) & ! Q2 * b2
                             + GMTR_t(im1j  ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AJ ,ht) & ! Q3 * b2
                             - GMTR_t(im1j  ,k0,l,TI,W2) * GMTR_a(im1j  ,k0,l,AI ,ht) & ! Q3 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W2) * GMTR_a(im1j  ,k0,l,AI ,ht) & ! Q4 * b3
                             - GMTR_t(im1jm1,k0,l,TJ,W2) * GMTR_a(im1jm1,k0,l,AIJ,ht) & ! Q4 * b4
                             - GMTR_t(ijm1  ,k0,l,TJ,W3) * GMTR_a(im1jm1,k0,l,AIJ,ht) & ! Q6 * b4
                             + GMTR_t(ijm1  ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AI ,ht) & ! Q6 * b6
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! ip1j
      coef_rot(ij,1,d,l) = ( - GMTR_t(ijm1  ,k0,l,TJ,W2) * GMTR_a(im1jm1,k0,l,AIJ,ht) & ! Q6 * b4
                             + GMTR_t(ijm1  ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AI ,ht) & ! Q6 * b6
                             + GMTR_t(ij    ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AI ,ht) & ! Q1 * b6
                             + GMTR_t(ij    ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AIJ,ht) & ! Q1 * b1
                           ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      ! ip1jp1
      coef_rot(ij,2,d,l) = ( + GMTR_t(ij    ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AI ,ht) & ! Q1 * b6
```

```
                          + GMTR_t(ij    ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AIJ,ht) & ! Q1 * b1
                          + GMTR_t(ij    ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AIJ,ht) & ! Q2 * b1
                          + GMTR_t(ij    ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AJ ,ht) & ! Q2 * b2
                        ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
        ! ijp1
        coef_rot(ij,3,d,l) = ( + GMTR_t(ij    ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AIJ,ht) & ! Q2 * b1
                               + GMTR_t(ij    ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AJ ,ht) & ! Q2 * b2
                               + GMTR_t(im1j  ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AJ ,ht) & ! Q3 * b2
                               - GMTR_t(im1j  ,k0,l,TI,W3) * GMTR_a(im1j  ,k0,l,AI ,ht) & ! Q3 * b3
                             ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
        ! im1j
        coef_rot(ij,4,d,l) = ( + GMTR_t(im1j  ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AJ ,ht) & ! Q3 * b2
                               - GMTR_t(im1j  ,k0,l,TI,W1) * GMTR_a(im1j  ,k0,l,AI ,ht) & ! Q3 * b3
                               - GMTR_t(im1jm1,k0,l,TJ,W3) * GMTR_a(im1j  ,k0,l,AI ,ht) & ! Q4 * b3
                               - GMTR_t(im1jm1,k0,l,TJ,W3) * GMTR_a(im1jm1,k0,l,AIJ,ht) & ! Q4 * b4
                             ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
        ! im1jm1
        coef_rot(ij,5,d,l) = ( - GMTR_t(im1jm1,k0,l,TJ,W1) * GMTR_a(im1j  ,k0,l,AI ,ht) & ! Q4 * b3
                               - GMTR_t(im1jm1,k0,l,TJ,W1) * GMTR_a(im1jm1,k0,l,AIJ,ht) & ! Q4 * b4
                             ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
        ! ijm1
        coef_rot(ij,6,d,l) = ( - GMTR_t(ijm1  ,k0,l,TJ,W1) * GMTR_a(im1jm1,k0,l,AIJ,ht) & ! Q6 * b4
                               + GMTR_t(ijm1  ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AI ,ht) & ! Q6 * b6
                             ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
        !$omp end master
     endif

  enddo ! loop d
  enddo ! loop l
  !$omp end parallel

```

This section is for the singular point. Note that `ADM_have_sgp(1)` is true.

The last section of this subroutine is for the pole region and is as follows.

```
  if ( ADM_have_pl ) then
     n = ADM_gslf_pl
     do l = 1, ADM_lall_pl
     do d = 1, ADM_nxyz
        ht = d + HTX - 1

        coef = 0.0_RP
        do v = ADM_gmin_pl, ADM_gmax_pl
           ij   = v
           ijp1 = v + 1
           if( ijp1 == ADM_gmax_pl+1 ) ijp1 = ADM_gmin_pl

           coef = coef + ( GMTR_t_pl(ij,k0,l,W1) * GMTR_a_pl(ij  ,k0,l,ht) &
                         + GMTR_t_pl(ij,k0,l,W1) * GMTR_a_pl(ijp1,k0,l,ht) )
        enddo
        coef_rot_pl(0,d,l) = coef * 0.5_RP * GMTR_p_pl(n,k0,l,P_RAREA)

        do v = ADM_gmin_pl, ADM_gmax_pl
           ij   = v
           ijp1 = v + 1
           ijm1 = v - 1
           if( ijp1 == ADM_gmax_pl + 1 ) ijp1 = ADM_gmin_pl
           if( ijm1 == ADM_gmin_pl - 1 ) ijm1 = ADM_gmax_pl

           coef_rot_pl(v-1,d,l) = ( + GMTR_t_pl(ijm1,k0,l,W3) * GMTR_a_pl(ijm1,k0,l,ht) &
                                    + GMTR_t_pl(ijm1,k0,l,W3) * GMTR_a_pl(ij  ,k0,l,ht) &
                                    + GMTR_t_pl(ij  ,k0,l,W2) * GMTR_a_pl(ij  ,k0,l,ht) &
                                    + GMTR_t_pl(ij  ,k0,l,W2) * GMTR_a_pl(ijp1,k0,l,ht) &
                                  ) * 0.5_RP * GMTR_p_pl(n,k0,l,P_RAREA)
        enddo
     enddo ! loop d
     enddo ! loop l
  endif

  return
end subroutine OPRT_rotation_setup
```

Note the data layout of coefficient is slightly different between the normal region and the pole region, First dimension of `coef_rot_pl` corresponds to the second dimention of `coef_rot`. The range of index `v` of the inner-most loop are `ADM_gmin_pl` and `ADM_gmax_pl`, which means the five grid points surrounding the pole point.

(6) OPRT_gradient_setup

This subroutine is also similar to the previous subroutines, but this one is for the gradient operator, subroutine OPRT_gradient.

Argument lists and local variables definition part of this subroutine is as follows.

```
subroutine OPRT_gradient_setup( &
     GMTR_p,    GMTR_p_pl,    &
     GMTR_t,    GMTR_t_pl,    &
     GMTR_a,    GMTR_a_pl,    &
     coef_grad, coef_grad_pl )
!ESC!    use mod_adm, only: &
!ESC!       ADM_have_pl,  &
!ESC!       ADM_have_sgp, &
!ESC!       ADM_gall_1d,  &
!ESC!       ADM_gmin,     &
!ESC!       ADM_gmax,     &
!ESC!       ADM_gslf_pl,  &
!ESC!       ADM_gmin_pl,  &
!ESC!       ADM_gmax_pl
!ESC!    use mod_gmtr, only: &
!ESC!       P_RAREA => GMTR_p_RAREA, &
!ESC!       W1      => GMTR_t_W1,    &
!ESC!       W2      => GMTR_t_W2,    &
!ESC!       W3      => GMTR_t_W3,    &
!ESC!       HNX     => GMTR_a_HNX,   &
!ESC!       GMTR_p_nmax,             &
!ESC!       GMTR_t_nmax,             &
!ESC!       GMTR_a_nmax,             &
!ESC!       GMTR_a_nmax_pl
  implicit none

  real(RP), intent(in)  :: GMTR_p      (ADM_gall   ,K0,ADM_lall   ,      GMTR_p_nmax   )
  real(RP), intent(in)  :: GMTR_p_pl   (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_p_nmax   )
  real(RP), intent(in)  :: GMTR_t      (ADM_gall   ,K0,ADM_lall   ,TI:TJ,GMTR_t_nmax   )
  real(RP), intent(in)  :: GMTR_t_pl   (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_t_nmax   )
  real(RP), intent(in)  :: GMTR_a      (ADM_gall   ,K0,ADM_lall   ,AI:AJ,GMTR_a_nmax   )
  real(RP), intent(in)  :: GMTR_a_pl   (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_a_nmax_pl)
  real(RP), intent(out) :: coef_grad   (ADM_gall,0:6        ,ADM_nxyz,ADM_lall    )
  real(RP), intent(out) :: coef_grad_pl(         0:ADM_vlink,ADM_nxyz,ADM_lall_pl)

  integer  :: gmin, gmax, iall, gall, nxyz, lall

  integer  :: ij
  integer  :: ip1j, ijp1, ip1jp1
  integer  :: im1j, ijm1, im1jm1

  real(RP) :: coef
  integer  :: g, l, d, n, v, hn
  !---------------------------------------------------------------------------

```

Input arguments are the same with subroutine OPRT_divergence_setup and OPRT_rotation_setup, the metrics calculated by the subroutines described before in this section. Output arguments coef_grad and coef_grad_pl are the coefficients for the gradient operator for the normal region and the pole region, respectively. The second dimension of coef_grad has the range 0:6, which corresponds to the 7-point stencil calculation. Also the second dimension of coef_grad_pl has the range 0:ADM_vlink where ADM_vlink is 5, corresponds to the 6-point stencil calculation. Note that the pole region is a pentagon.

The procedure of this subroutine is consist of three parts. The first part is as follows.

```
  !if( IO_L ) write(IO_FID_LOG,*) '*** setup coefficient of gradient operator'

  gmin = (ADM_gmin-1)*ADM_gall_1d + ADM_gmin
  gmax = (ADM_gmax-1)*ADM_gall_1d + ADM_gmax
  iall = ADM_gall_1d
  gall = ADM_gall
  nxyz = ADM_nxyz
  lall = ADM_lall

  !$omp parallel workshare
  coef_grad   (:,:,:,:) = 0.0_RP
  !$omp end parallel workshare
  coef_grad_pl(  :,:,:) = 0.0_RP

```

```fortran
    !$omp parallel default(none),private(g,d,l,hn,ij,ip1j,ip1jp1,ijp1,im1j,ijm1,im1jm1), &
    !$omp shared(ADM_have_sgp,gmin,gmax,iall,gall,nxyz,lall,coef_grad,GMTR_p,GMTR_t,GMTR_a)
    do l = 1, lall
    do d = 1, nxyz
       hn = d + HNX - 1

       !$omp do
       do g = gmin, gmax
          ij     = g
          ip1j   = g + 1
          ip1jp1 = g + iall + 1
          ijp1   = g + iall
          im1j   = g - 1
          im1jm1 = g - iall - 1
          ijm1   = g - iall

          ! ij
          coef_grad(ij,0,d,l) = ( + GMTR_t(ij    ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q1 * b6
                                  + GMTR_t(ij    ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q1 * b1
                                  + GMTR_t(ij    ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q2 * b1
                                  + GMTR_t(ij    ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q2 * b2
                                  + GMTR_t(im1j  ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q3 * b2
                                  - GMTR_t(im1j  ,k0,l,TI,W2) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q3 * b3
                                  - GMTR_t(im1jm1,k0,l,TJ,W2) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q4 * b3
                                  - GMTR_t(im1jm1,k0,l,TI,W3) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q4 * b4
                                  - GMTR_t(im1jm1,k0,l,TJ,W2) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q5 * b4
                                  - GMTR_t(im1jm1,k0,l,TI,W3) * GMTR_a(ijm1  ,k0,l,AJ ,hn) & ! Q5 * b5
                                  - GMTR_t(ijm1  ,k0,l,TJ,W3) * GMTR_a(ijm1  ,k0,l,AJ ,hn) & ! Q6 * b5
                                  + GMTR_t(ijm1  ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q6 * b6
                                  - 2.0_RP * GMTR_a(ij    ,k0,l,AIJ,hn)                    & ! P0 * b1
                                  - 2.0_RP * GMTR_a(ij    ,k0,l,AJ ,hn)                    & ! P0 * b2
                                  + 2.0_RP * GMTR_a(im1j  ,k0,l,AI ,hn)                    & ! P0 * b3
                                  + 2.0_RP * GMTR_a(im1jm1,k0,l,AIJ,hn)                    & ! P0 * b4
                                  + 2.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,hn)                    & ! P0 * b5
                                  - 2.0_RP * GMTR_a(ij    ,k0,l,AI ,hn)                    & ! P0 * b6
                                ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
          ! ip1j
          coef_grad(ij,1,d,l) = ( - GMTR_t(ijm1  ,k0,l,TJ,W2) * GMTR_a(ijm1  ,k0,l,AJ ,hn) & ! Q6 * b5
                                  + GMTR_t(ijm1  ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q6 * b6
                                  + GMTR_t(ij    ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q1 * b6
                                  + GMTR_t(ij    ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q1 * b1
                                ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
          ! ip1jp1
          coef_grad(ij,2,d,l) = ( + GMTR_t(ij    ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q1 * b6
                                  + GMTR_t(ij    ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q1 * b1
                                  + GMTR_t(ij    ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q2 * b1
                                  + GMTR_t(ij    ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q2 * b2
                                ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
          ! ijp1
          coef_grad(ij,3,d,l) = ( + GMTR_t(ij    ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q2 * b1
                                  + GMTR_t(ij    ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q2 * b2
                                  + GMTR_t(im1j  ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q3 * b2
                                  - GMTR_t(im1j  ,k0,l,TI,W3) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q3 * b3
                                ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
          ! im1j
          coef_grad(ij,4,d,l) = ( + GMTR_t(im1j  ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q3 * b2
                                  - GMTR_t(im1j  ,k0,l,TI,W1) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q3 * b3
                                  - GMTR_t(im1jm1,k0,l,TJ,W3) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q4 * b3
                                  - GMTR_t(im1jm1,k0,l,TJ,W3) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q4 * b4
                                ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
          ! im1jm1
          coef_grad(ij,5,d,l) = ( - GMTR_t(im1jm1,k0,l,TJ,W1) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q4 * b3
                                  - GMTR_t(im1jm1,k0,l,TJ,W1) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q4 * b4
                                  - GMTR_t(im1jm1,k0,l,TI,W1) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q5 * b4
                                  - GMTR_t(im1jm1,k0,l,TI,W1) * GMTR_a(ijm1  ,k0,l,AJ ,hn) & ! Q5 * b5
                                ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
          ! ijm1
          coef_grad(ij,6,d,l) = ( - GMTR_t(im1jm1,k0,l,TI,W2) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q5 * b4
                                  - GMTR_t(im1jm1,k0,l,TI,W2) * GMTR_a(ijm1  ,k0,l,AJ ,hn) & ! Q5 * b5
                                  - GMTR_t(ijm1  ,k0,l,TJ,W1) * GMTR_a(ijm1  ,k0,l,AJ ,hn) & ! Q6 * b5
                                  + GMTR_t(ijm1  ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q6 * b6
                                ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
       enddo
       !$omp end do

```

The first part above and the second part below are in a long l-loop. In the g-loop represents horizontal index, 7 coefficients are calculated separately from the various metrics.

The second part is as follows.

```
      if ( ADM_have_sgp(l) ) then ! pentagon
         !$omp master
         ij     = gmin
         ip1j   = gmin + 1
         ip1jp1 = gmin + iall + 1
         ijp1   = gmin + iall
         im1j   = gmin - 1
         im1jm1 = gmin - iall - 1
         ijm1   = gmin - iall

         ! ij
         coef_grad(ij,0,d,l) = ( + GMTR_t(ij    ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q1 * b6
                                 + GMTR_t(ij    ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q1 * b1
                                 + GMTR_t(ij    ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q2 * b1
                                 + GMTR_t(ij    ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q2 * b2
                                 + GMTR_t(im1j  ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q3 * b2
                                 - GMTR_t(im1j  ,k0,l,TI,W2) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q3 * b3
                                 - GMTR_t(im1jm1,k0,l,TJ,W2) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q4 * b3
                                 - GMTR_t(im1jm1,k0,l,TJ,W2) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q4 * b4
                                 - GMTR_t(ijm1  ,k0,l,TJ,W3) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q6 * b4
                                 + GMTR_t(ijm1  ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q6 * b6
                                 - 2.0_RP * GMTR_a(ij    ,k0,l,AIJ,hn)                    & ! P0 * b1
                                 - 2.0_RP * GMTR_a(ij    ,k0,l,AJ ,hn)                    & ! P0 * b2
                                 + 2.0_RP * GMTR_a(im1j  ,k0,l,AI ,hn)                    & ! P0 * b3
                                 + 2.0_RP * GMTR_a(im1jm1,k0,l,AIJ,hn)                    & ! P0 * b4
                                 - 2.0_RP * GMTR_a(ij    ,k0,l,AI ,hn)                    & ! P0 * b6
                               ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
         ! ip1j
         coef_grad(ij,1,d,l) = ( - GMTR_t(ijm1  ,k0,l,TJ,W2) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q6 * b4
                                 + GMTR_t(ijm1  ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q6 * b6
                                 + GMTR_t(ij    ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q1 * b6
                                 + GMTR_t(ij    ,k0,l,TI,W2) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q1 * b1
                               ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
         ! ip1jp1
         coef_grad(ij,2,d,l) = ( + GMTR_t(ij    ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q1 * b6
                                 + GMTR_t(ij    ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q1 * b1
                                 + GMTR_t(ij    ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q2 * b1
                                 + GMTR_t(ij    ,k0,l,TJ,W2) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q2 * b2
                               ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
         ! ijp1
         coef_grad(ij,3,d,l) = ( + GMTR_t(ij    ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AIJ,hn) & ! Q2 * b1
                                 + GMTR_t(ij    ,k0,l,TJ,W3) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q2 * b2
                                 + GMTR_t(im1j  ,k0,l,TI,W3) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q3 * b2
                                 - GMTR_t(im1j  ,k0,l,TI,W3) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q3 * b3
                               ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
         ! im1j
         coef_grad(ij,4,d,l) = ( + GMTR_t(im1j  ,k0,l,TI,W1) * GMTR_a(ij    ,k0,l,AJ ,hn) & ! Q3 * b2
                                 - GMTR_t(im1j  ,k0,l,TI,W1) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q3 * b3
                                 - GMTR_t(im1jm1,k0,l,TJ,W3) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q4 * b3
                                 - GMTR_t(im1jm1,k0,l,TJ,W3) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q4 * b4
                               ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
         ! im1jm1
         coef_grad(ij,5,d,l) = ( - GMTR_t(im1jm1,k0,l,TJ,W1) * GMTR_a(im1j  ,k0,l,AI ,hn) & ! Q4 * b3
                                 - GMTR_t(im1jm1,k0,l,TJ,W1) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q4 * b4
                               ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
         ! ijm1
         coef_grad(ij,6,d,l) = ( - GMTR_t(ijm1  ,k0,l,TJ,W1) * GMTR_a(im1jm1,k0,l,AIJ,hn) & ! Q6 * b4
                                 + GMTR_t(ijm1  ,k0,l,TJ,W1) * GMTR_a(ij    ,k0,l,AI ,hn) & ! Q6 * b6
                               ) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
         !$omp end master
      endif

   enddo ! loop d
   enddo ! loop l
   !$omp end parallel

```

This section is for the singular point. Note that `ADM_have_sgp(l)` is true.

The last section of this subroutine is for the pole region and is as follows.

```
   if ( ADM_have_pl ) then
      n = ADM_gslf_pl

      do l = 1, ADM_lall_pl
      do d = 1, ADM_nxyz
         hn = d + HNX - 1

         coef = 0.0_RP
```

```
        do v = ADM_gmin_pl, ADM_gmax_pl
           ij   = v
           ijp1 = v + 1
           if( ijp1 == ADM_gmax_pl + 1 ) ijp1 = ADM_gmin_pl

           coef = coef + 2.0_RP * ( GMTR_t_pl(ij,k0,l,W1) - 1.0_RP ) * GMTR_a_pl(ijp1,k0,l,hn)
        enddo
        coef_grad_pl(0,d,l) = coef * 0.5_RP * GMTR_p_pl(n,k0,l,P_RAREA)

        do v = ADM_gmin_pl, ADM_gmax_pl
           ij   = v
           ijp1 = v + 1
           ijm1 = v - 1
           if( ijp1 == ADM_gmax_pl + 1 ) ijp1 = ADM_gmin_pl
           if( ijm1 == ADM_gmin_pl - 1 ) ijm1 = ADM_gmax_pl

           coef_grad_pl(v-1,d,l) = ( + GMTR_t_pl(ijm1,k0,l,W3) * GMTR_a_pl(ijm1,k0,l,hn) &
                                     + GMTR_t_pl(ijm1,k0,l,W3) * GMTR_a_pl(ij   ,k0,l,hn) &
                                     + GMTR_t_pl(ij   ,k0,l,W2) * GMTR_a_pl(ij   ,k0,l,hn) &
                                     + GMTR_t_pl(ij   ,k0,l,W2) * GMTR_a_pl(ijp1,k0,l,hn) &
                                   ) * 0.5_RP * GMTR_p_pl(n,k0,l,P_RAREA)
        enddo
     enddo ! loop d
     enddo ! loop l
  endif

  return
end subroutine OPRT_gradient_setup
```

Note the data layout of coefficient is slightly different between the normal region and the pole region, First dimension of `coef_grad_pl` corresponds to the second dimention of `coef_grad`. The range of index `v` of the inner-most loop are `ADM_gmin_pl` and `ADM_gmax_pl`, which means the five grid points surrounding the pole point.

(7) `OPRT_laplacian_setup`

This subroutine is also similar to the previous three subroutines, but this one is for the lapracian operator, subroutine `OPRT_laplacian`.

Argument lists and local variables definition part of this subroutine is as follows.

```
subroutine OPRT_laplacian_setup( &
     GMTR_p,   GMTR_p_pl,  &
     GMTR_t,   GMTR_t_pl,  &
     GMTR_a,   GMTR_a_pl,  &
     coef_lap, coef_lap_pl )
!ESC!    use mod_adm, only: &
!ESC!       ADM_have_pl,  &
!ESC!       ADM_have_sgp, &
!ESC!       ADM_gall_1d,  &
!ESC!       ADM_gmin,     &
!ESC!       ADM_gmax,     &
!ESC!       ADM_gslf_pl,  &
!ESC!       ADM_gmin_pl,  &
!ESC!       ADM_gmax_pl
!ESC!    use mod_gmtr, only: &
!ESC!       P_RAREA => GMTR_p_RAREA, &
!ESC!       T_RAREA => GMTR_t_RAREA, &
!ESC!       HNX     => GMTR_a_HNX,   &
!ESC!       TNX     => GMTR_a_TNX,   &
!ESC!       TN2X    => GMTR_a_TN2X,  &
!ESC!       GMTR_p_nmax,             &
!ESC!       GMTR_t_nmax,             &
!ESC!       GMTR_a_nmax,             &
!ESC!       GMTR_a_nmax_pl
  implicit none

  real(RP), intent(in)  :: GMTR_p     (ADM_gall    ,K0,ADM_lall    ,      GMTR_p_nmax   )
  real(RP), intent(in)  :: GMTR_p_pl  (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_p_nmax   )
  real(RP), intent(in)  :: GMTR_t     (ADM_gall    ,K0,ADM_lall    ,TI:TJ,GMTR_t_nmax   )
  real(RP), intent(in)  :: GMTR_t_pl  (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_t_nmax   )
  real(RP), intent(in)  :: GMTR_a     (ADM_gall    ,K0,ADM_lall    ,AI:AJ,GMTR_a_nmax   )
  real(RP), intent(in)  :: GMTR_a_pl  (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_a_nmax_pl)
  real(RP), intent(out) :: coef_lap   (ADM_gall,0:6        ,ADM_lall    )
  real(RP), intent(out) :: coef_lap_pl(         0:ADM_vlink,ADM_lall_pl)
```

```
    integer  :: gmin, gmax, iall, gall, nxyz, lall

    integer  :: ij
    integer  :: ip1j, ijp1, ip1jp1
    integer  :: im1j, ijm1, im1jm1

    integer  :: g, l, d, n, v, hn, tn, tn2
    !---------------------------------------------------------------------------

```

Input arguments are the same with the previous three subroutines, the metrics calculated by the subroutines described before in this section. But the output argument `coef_lap` and `coef_lap_pl` have the different shape with the previous ones, such as `coef_div` and `coef_div_pl`. Since the laplacian operator is the second order differentiation, different from the divergence, rotation, gradient, these are the first order, and the result is a scalar, not a vector, there are no need to specify the 3-D direction, which are the third dimension of such as `coef_div`. Instead of that, calculation of the coefficient must include the contribution of more grid points than the operator of the first order differentiation.

The procedure of this subroutine is consist of three parts. The first part is as follows.

```
    !if( IO_L ) write(IO_FID_LOG,*) '*** setup coefficient of laplacian operator'

    gmin = (ADM_gmin-1)*ADM_gall_1d + ADM_gmin
    gmax = (ADM_gmax-1)*ADM_gall_1d + ADM_gmax
    iall = ADM_gall_1d
    gall = ADM_gall
    nxyz = ADM_nxyz
    lall = ADM_lall

    !$omp parallel workshare
    coef_lap   (:,:,:) = 0.0_RP
    !$omp end parallel workshare
    coef_lap_pl(  :,:) = 0.0_RP

    !$omp parallel default(none),private(g,d,l,hn,tn,ij,ip1j,ip1jp1,ijp1,im1j,ijm1,im1jm1), &
    !$omp shared(ADM_have_sgp,gmin,gmax,iall,gall,nxyz,lall,coef_lap,GMTR_p,GMTR_t,GMTR_a)
    do l = 1, lall

       do d = 1, nxyz
          hn = d + HNX - 1
          tn = d + TNX - 1

          !$omp do
          do g = gmin, gmax
             ij     = g
             ip1j   = g + 1
             ip1jp1 = g + iall + 1
             ijp1   = g + iall
             im1j   = g - 1
             im1jm1 = g - iall - 1
             ijm1   = g - iall

             ! ij
             coef_lap(ij,0,l) = coef_lap(ij,0,l) &
                              + GMTR_t(ij    ,k0,l,TI,T_RAREA) &
                              * ( - 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                  + 2.0_RP * GMTR_a(ip1j  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                  + 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                  - 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                  + 2.0_RP * GMTR_a(ip1j  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                  + 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) )

             coef_lap(ij,0,l) = coef_lap(ij,0,l) &
                              + GMTR_t(ij    ,k0,l,TJ,T_RAREA) &
                              * ( - 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                  - 2.0_RP * GMTR_a(ijp1  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                  + 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                  - 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                  - 2.0_RP * GMTR_a(ijp1  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                  + 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) )

             coef_lap(ij,0,l) = coef_lap(ij,0,l) &
                              + GMTR_t(im1j  ,k0,l,TI,T_RAREA) &
                              * ( - 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                  - 2.0_RP * GMTR_a(im1j  ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                  - 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                  + 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
```

```
                                 + 2.0_RP * GMTR_a(im1j  ,k0,l,AIJ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 + 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) )

            coef_lap(ij,0,l) = coef_lap(ij,0,l) &
                             + GMTR_t(im1jm1,k0,l,TJ,T_RAREA) &
                             * ( - 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 + 2.0_RP * GMTR_a(im1jm1,k0,l,AJ ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 + 1.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 - 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                                 + 2.0_RP * GMTR_a(im1jm1,k0,l,AJ ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                                 + 1.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) )

            coef_lap(ij,0,l) = coef_lap(ij,0,l) &
                             + GMTR_t(im1jm1,k0,l,TI,T_RAREA) &
                             * ( - 1.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                                 - 2.0_RP * GMTR_a(im1jm1,k0,l,AI ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                                 + 1.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                                 - 1.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                                 - 2.0_RP * GMTR_a(im1jm1,k0,l,AI ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                                 + 1.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) )

            coef_lap(ij,0,l) = coef_lap(ij,0,l) &
                             + GMTR_t(ijm1  ,k0,l,TJ,T_RAREA) &
                             * ( - 1.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                                 - 2.0_RP * GMTR_a(ijm1  ,k0,l,AIJ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                                 - 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                                 + 1.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 + 2.0_RP * GMTR_a(ijm1  ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 + 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) )

            ! ip1j
            coef_lap(ij,1,l) = coef_lap(ij,1,l) &
                             + GMTR_t(ijm1  ,k0,l,TJ,T_RAREA) &
                             * ( - 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                                 + 2.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                                 + 1.0_RP * GMTR_a(ijm1  ,k0,l,AIJ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                                 + 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 - 2.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 - 1.0_RP * GMTR_a(ijm1  ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) )

            coef_lap(ij,1,l) = coef_lap(ij,1,l) &
                             + GMTR_t(ij    ,k0,l,TI,T_RAREA) &
                             * ( - 1.0_RP * GMTR_a(ip1j  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 - 2.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 - 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 - 1.0_RP * GMTR_a(ip1j  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 - 2.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 - 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) )

            ! ip1jp1
            coef_lap(ij,2,l) = coef_lap(ij,2,l) &
                             + GMTR_t(ij    ,k0,l,TI,T_RAREA) &
                             * ( + 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 + 2.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 - 1.0_RP * GMTR_a(ip1j  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 + 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 + 2.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 - 1.0_RP * GMTR_a(ip1j  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) )

            coef_lap(ij,2,l) = coef_lap(ij,2,l) &
                             + GMTR_t(ij    ,k0,l,TJ,T_RAREA) &
                             * ( + 1.0_RP * GMTR_a(ijp1  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 - 2.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 - 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 + 1.0_RP * GMTR_a(ijp1  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 - 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 - 2.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) )

            ! ijp1
            coef_lap(ij,3,l) = coef_lap(ij,3,l) &
                             + GMTR_t(ij    ,k0,l,TJ,T_RAREA) &
                             * ( + 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 + 2.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 + 1.0_RP * GMTR_a(ijp1  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 + 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 + 2.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 + 1.0_RP * GMTR_a(ijp1  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) )

            coef_lap(ij,3,l) = coef_lap(ij,3,l) &
                             + GMTR_t(im1j  ,k0,l,TI,T_RAREA) &
                             * ( + 1.0_RP * GMTR_a(im1j  ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 + 2.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
```

```
                         - 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                         - 1.0_RP * GMTR_a(im1j  ,k0,l,AIJ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                         - 2.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                         + 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) )

          ! im1j
          coef_lap(ij,4,l) = coef_lap(ij,4,l) &
                           + GMTR_t(im1j  ,k0,l,TI,T_RAREA) &
                           * ( - 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                               + 2.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                               + 1.0_RP * GMTR_a(im1j  ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                               + 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               - 2.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               - 1.0_RP * GMTR_a(im1j  ,k0,l,AIJ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) )

          coef_lap(ij,4,l) = coef_lap(ij,4,l) &
                           + GMTR_t(im1jm1,k0,l,TJ,T_RAREA) &
                           * ( - 1.0_RP * GMTR_a(im1jm1,k0,l,AJ ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               - 2.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               - 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               - 1.0_RP * GMTR_a(im1jm1,k0,l,AJ ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               - 2.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               - 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) )

          ! im1jm1
          coef_lap(ij,5,l) = coef_lap(ij,5,l) &
                           + GMTR_t(im1jm1,k0,l,TJ,T_RAREA) &
                           * ( + 1.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               + 2.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               - 1.0_RP * GMTR_a(im1jm1,k0,l,AJ ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               + 1.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               + 2.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               - 1.0_RP * GMTR_a(im1jm1,k0,l,AJ ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) )

          coef_lap(ij,5,l) = coef_lap(ij,5,l) &
                           + GMTR_t(im1jm1,k0,l,TI,T_RAREA) &
                           * ( + 1.0_RP * GMTR_a(im1jm1,k0,l,AI ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               - 2.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               - 1.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               + 1.0_RP * GMTR_a(im1jm1,k0,l,AI ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                               - 2.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                               - 1.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) )

          ! ijm1
          coef_lap(ij,6,l) = coef_lap(ij,6,l) &
                           + GMTR_t(im1jm1,k0,l,TI,T_RAREA) &
                           * ( + 1.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               + 2.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               + 1.0_RP * GMTR_a(im1jm1,k0,l,AI ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               + 1.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                               + 2.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                               + 1.0_RP * GMTR_a(im1jm1,k0,l,AI ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) )

          coef_lap(ij,6,l) = coef_lap(ij,6,l) &
                           + GMTR_t(ijm1  ,k0,l,TJ,T_RAREA) &
                           * ( + 1.0_RP * GMTR_a(ijm1  ,k0,l,AIJ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                               + 2.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                               - 1.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(ijm1  ,k0,l,AJ ,hn) &
                               - 1.0_RP * GMTR_a(ijm1  ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                               - 2.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                               + 1.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) )
       enddo
       !$omp end do
    enddo ! loop d

```

The first part above and the second part below are in a long l-loop. In the g-loop represents horizontal index, 7 coefficients are calculated separately from the various metrics. Note that each 7 coefficients contains contribution from more grid points than previous operator subroutines.

The second part is as follows.

```
    if ( ADM_have_sgp(l) ) then ! pentagon
       !$omp master
       coef_lap(gmin,0,l) = 0.0_RP
       coef_lap(gmin,1,l) = 0.0_RP
       coef_lap(gmin,2,l) = 0.0_RP
       coef_lap(gmin,3,l) = 0.0_RP
       coef_lap(gmin,4,l) = 0.0_RP
```

```
       coef_lap(gmin,5,l) = 0.0_RP
       coef_lap(gmin,6,l) = 0.0_RP

       do d = 1, ADM_nxyz
          hn = d + HNX - 1
          tn = d + TNX - 1

          ij     = gmin
          ip1j   = gmin + 1
          ip1jp1 = gmin + iall + 1
          ijp1   = gmin + iall
          im1j   = gmin - 1
          im1jm1 = gmin - iall - 1
          ijm1   = gmin - iall

          ! ij
          coef_lap(ij,0,l) = coef_lap(ij,0,l) &
                           + GMTR_t(ij    ,k0,l,TI,T_RAREA) &
                           * ( - 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                               + 2.0_RP * GMTR_a(ip1j  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                               + 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                               - 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                               + 2.0_RP * GMTR_a(ip1j  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                               + 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) )

          coef_lap(ij,0,l) = coef_lap(ij,0,l) &
                           + GMTR_t(ij    ,k0,l,TJ,T_RAREA) &
                           * ( - 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                               - 2.0_RP * GMTR_a(ijp1  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                               + 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                               - 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                               - 2.0_RP * GMTR_a(ijp1  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                               + 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) )

          coef_lap(ij,0,l) = coef_lap(ij,0,l) &
                           + GMTR_t(im1j  ,k0,l,TI,T_RAREA) &
                           * ( - 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                               - 2.0_RP * GMTR_a(im1j  ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                               - 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                               + 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               + 2.0_RP * GMTR_a(im1j  ,k0,l,AIJ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               + 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) )

          coef_lap(ij,0,l) = coef_lap(ij,0,l) &
                           + GMTR_t(im1jm1,k0,l,TJ,T_RAREA) &
                           * ( - 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               + 2.0_RP * GMTR_a(im1jm1,k0,l,AJ ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               + 1.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                               - 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               + 2.0_RP * GMTR_a(im1jm1,k0,l,AJ ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               + 1.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) )

          coef_lap(ij,0,l) = coef_lap(ij,0,l) &
                           + GMTR_t(ijm1  ,k0,l,TJ,T_RAREA) &
                           * ( - 1.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               - 2.0_RP * GMTR_a(ijm1  ,k0,l,AIJ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               - 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               + 1.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                               + 2.0_RP * GMTR_a(ijm1  ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                               + 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) )

          ! ip1j
          coef_lap(ij,1,l) = coef_lap(ij,1,l) &
                           + GMTR_t(ijm1  ,k0,l,TJ,T_RAREA) &
                           * ( + 1.0_RP * GMTR_a(ijm1  ,k0,l,AIJ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               + 2.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               - 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                               - 1.0_RP * GMTR_a(ijm1  ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                               - 2.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                               + 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) )

          coef_lap(ij,1,l) = coef_lap(ij,1,l) &
                           + GMTR_t(ij    ,k0,l,TI,T_RAREA) &
                           * ( - 1.0_RP * GMTR_a(ip1j  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                               - 2.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                               - 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                               - 1.0_RP * GMTR_a(ip1j  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                               - 2.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                               - 1.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) )

          ! ip1jp1
          coef_lap(ij,2,l) = coef_lap(ij,2,l) &
```

```
                             + GMTR_t(ij    ,k0,l,TI,T_RAREA) &
                             * ( + 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 + 2.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 - 1.0_RP * GMTR_a(ip1j  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 + 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 + 2.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 - 1.0_RP * GMTR_a(ip1j  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) )

             coef_lap(ij,2,l) = coef_lap(ij,2,l) &
                             + GMTR_t(ij    ,k0,l,TJ,T_RAREA) &
                             * ( + 1.0_RP * GMTR_a(ijp1  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 - 2.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 - 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 + 1.0_RP * GMTR_a(ijp1  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 - 2.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 - 1.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) )

             ! ijp1
             coef_lap(ij,3,l) = coef_lap(ij,3,l) &
                             + GMTR_t(ij    ,k0,l,TJ,T_RAREA) &
                             * ( + 1.0_RP * GMTR_a(ijp1  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 + 2.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 + 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AIJ,hn) &
                                 + 1.0_RP * GMTR_a(ijp1  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 + 2.0_RP * GMTR_a(ij    ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 + 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) )

             coef_lap(ij,3,l) = coef_lap(ij,3,l) &
                             + GMTR_t(im1j  ,k0,l,TI,T_RAREA) &
                             * ( + 1.0_RP * GMTR_a(im1j  ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 + 2.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 - 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 - 1.0_RP * GMTR_a(im1j  ,k0,l,AIJ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 - 2.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 + 1.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) )

             ! im1j
             coef_lap(ij,4,l) = coef_lap(ij,4,l) &
                             + GMTR_t(im1j  ,k0,l,TI,T_RAREA) &
                             * ( + 1.0_RP * GMTR_a(im1j  ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 + 2.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 - 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AJ ,hn) &
                                 - 1.0_RP * GMTR_a(im1j  ,k0,l,AIJ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 - 2.0_RP * GMTR_a(ij    ,k0,l,AJ ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 + 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) )

             coef_lap(ij,4,l) = coef_lap(ij,4,l) &
                             + GMTR_t(im1jm1,k0,l,TJ,T_RAREA) &
                             * ( - 1.0_RP * GMTR_a(im1jm1,k0,l,AJ ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 - 2.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 - 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 - 1.0_RP * GMTR_a(im1jm1,k0,l,AJ ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                                 - 2.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                                 - 1.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) )

             ! im1jm1
             coef_lap(ij,5,l) = coef_lap(ij,5,l) &
                             + GMTR_t(im1jm1,k0,l,TJ,T_RAREA) &
                             * ( - 1.0_RP * GMTR_a(im1jm1,k0,l,AJ ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 + 2.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 + 1.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1j  ,k0,l,AI ,hn) &
                                 - 1.0_RP * GMTR_a(im1jm1,k0,l,AJ ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                                 + 2.0_RP * GMTR_a(im1j  ,k0,l,AI ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                                 + 1.0_RP * GMTR_a(im1jm1,k0,l,AIJ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) )

             ! ijm1
             coef_lap(ij,6,l) = coef_lap(ij,6,l) &
                             + GMTR_t(ijm1  ,k0,l,TJ,T_RAREA) &
                             * ( + 1.0_RP * GMTR_a(ijm1  ,k0,l,AIJ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                                 + 2.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                                 - 1.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(im1jm1,k0,l,AIJ,hn) &
                                 - 1.0_RP * GMTR_a(ijm1  ,k0,l,AIJ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 - 2.0_RP * GMTR_a(ij    ,k0,l,AI ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) &
                                 + 1.0_RP * GMTR_a(ijm1  ,k0,l,AJ ,tn) * GMTR_a(ij    ,k0,l,AI ,hn) )
          enddo ! loop d
          !$omp end master
       endif

       !$omp do
       do g = 1, gall
          coef_lap(g,0,l) = coef_lap(g,0,l) * GMTR_p(g,k0,l,P_RAREA) / 12.0_RP
          coef_lap(g,1,l) = coef_lap(g,1,l) * GMTR_p(g,k0,l,P_RAREA) / 12.0_RP
```

```
        coef_lap(g,2,l) = coef_lap(g,2,l) * GMTR_p(g,k0,l,P_RAREA) / 12.0_RP
        coef_lap(g,3,l) = coef_lap(g,3,l) * GMTR_p(g,k0,l,P_RAREA) / 12.0_RP
        coef_lap(g,4,l) = coef_lap(g,4,l) * GMTR_p(g,k0,l,P_RAREA) / 12.0_RP
        coef_lap(g,5,l) = coef_lap(g,5,l) * GMTR_p(g,k0,l,P_RAREA) / 12.0_RP
        coef_lap(g,6,l) = coef_lap(g,6,l) * GMTR_p(g,k0,l,P_RAREA) / 12.0_RP
     enddo
     !$omp end do

  enddo ! loop l
  !$omp end parallel

```

This section is for the singular point. Note that `ADM_have_sgp(1)` is true.

The last section of this subroutine is for the pole region and is as follows.

```
  if ( ADM_have_pl ) then
     n  = ADM_gslf_pl

     do l = 1,ADM_lall_pl

        do d = 1, ADM_nxyz
           hn  = d + HNX  - 1
           tn  = d + TNX  - 1
           tn2 = d + TN2X - 1

           do v = ADM_gmin_pl, ADM_gmax_pl
              ij   = v
              ijp1 = v + 1
              ijm1 = v - 1
              if( ijp1 == ADM_gmax_pl + 1 ) ijp1 = ADM_gmin_pl
              if( ijm1 == ADM_gmin_pl - 1 ) ijm1 = ADM_gmax_pl

              coef_lap_pl(0,l) = coef_lap_pl(0,l) &
                               + GMTR_t_pl(ijm1,k0,l,T_RAREA) &
                               * ( + 1.0_RP * GMTR_a_pl(ijm1,k0,l,tn ) * GMTR_a_pl(ij,k0,l,hn) &
                                   - 2.0_RP * GMTR_a_pl(ijm1,k0,l,tn2) * GMTR_a_pl(ij,k0,l,hn) &
                                   - 1.0_RP * GMTR_a_pl(ij  ,k0,l,tn ) * GMTR_a_pl(ij,k0,l,hn) )

              coef_lap_pl(0,l) = coef_lap_pl(0,l) &
                               + GMTR_t_pl(ij  ,k0,l,T_RAREA) &
                               * ( + 1.0_RP * GMTR_a_pl(ij  ,k0,l,tn ) * GMTR_a_pl(ij,k0,l,hn) &
                                   - 2.0_RP * GMTR_a_pl(ij  ,k0,l,tn2) * GMTR_a_pl(ij,k0,l,hn) &
                                   - 1.0_RP * GMTR_a_pl(ijp1,k0,l,tn ) * GMTR_a_pl(ij,k0,l,hn) )
           enddo

           do v = ADM_gmin_pl, ADM_gmax_pl
              ij   = v
              ijp1 = v + 1
              ijm1 = v - 1
              if( ijp1 == ADM_gmax_pl + 1 ) ijp1 = ADM_gmin_pl
              if( ijm1 == ADM_gmin_pl - 1 ) ijm1 = ADM_gmax_pl

              coef_lap_pl(v-1,l) = coef_lap_pl(v-1,l) &
                                 + GMTR_t_pl(ijm1,k0,l,T_RAREA) &
                                 * ( - 2.0_RP * GMTR_a_pl(ijm1,k0,l,tn ) * GMTR_a_pl(ijm1,k0,l,hn) &
                                     + 1.0_RP * GMTR_a_pl(ijm1,k0,l,tn2) * GMTR_a_pl(ijm1,k0,l,hn) &
                                     - 1.0_RP * GMTR_a_pl(ij  ,k0,l,tn ) * GMTR_a_pl(ijm1,k0,l,hn) &
                                     - 2.0_RP * GMTR_a_pl(ijm1,k0,l,tn ) * GMTR_a_pl(ij  ,k0,l,hn) &
                                     + 1.0_RP * GMTR_a_pl(ijm1,k0,l,tn2) * GMTR_a_pl(ij  ,k0,l,hn) &
                                     - 1.0_RP * GMTR_a_pl(ij  ,k0,l,tn ) * GMTR_a_pl(ij  ,k0,l,hn) )

              coef_lap_pl(v-1,l) = coef_lap_pl(v-1,l) &
                                 + GMTR_t_pl(ij  ,k0,l,T_RAREA) &
                                 * ( + 1.0_RP * GMTR_a_pl(ij  ,k0,l,tn ) * GMTR_a_pl(ij  ,k0,l,hn) &
                                     + 1.0_RP * GMTR_a_pl(ij  ,k0,l,tn2) * GMTR_a_pl(ij  ,k0,l,hn) &
                                     + 2.0_RP * GMTR_a_pl(ijp1,k0,l,tn ) * GMTR_a_pl(ij  ,k0,l,hn) &
                                     + 1.0_RP * GMTR_a_pl(ij  ,k0,l,tn ) * GMTR_a_pl(ijp1,k0,l,hn) &
                                     + 1.0_RP * GMTR_a_pl(ij  ,k0,l,tn2) * GMTR_a_pl(ijp1,k0,l,hn) &
                                     + 2.0_RP * GMTR_a_pl(ijp1,k0,l,tn ) * GMTR_a_pl(ijp1,k0,l,hn) )
           enddo
        enddo ! d loop

        do v = ADM_gslf_pl, ADM_gmax_pl
           coef_lap_pl(v-1,l) = coef_lap_pl(v-1,l) * GMTR_p_pl(n,k0,l,P_RAREA) / 12.0_RP
        enddo

     enddo ! l loop
  endif

```

```
  return
end subroutine OPRT_laplacian_setup
```

Note the data layout of coefficient is slightly different between the normal region and the pole region, First dimension of `coef_lap_pl` corresponds to the second dimention of `coef_lap`. The range of index `v` of the inner-most loop are `ADM_gmin_pl` and `ADM_gmax_pl`, which means the five grid points surrounding the pole point.

(8) `OPRT_diffusion_setup`

This subroutine is to calculate the coefficients for the diffusion operator, subroutine `OPRT_diffusion`, which is used in the `dyn_diffusion` kernel program.

Argument lists and local variables definition part of this subroutine is as follows.

```
subroutine OPRT_diffusion_setup( &
     GMTR_p,    GMTR_p_pl,    &
     GMTR_t,    GMTR_t_pl,    &
     GMTR_a,    GMTR_a_pl,    &
     coef_intp, coef_intp_pl, &
     coef_diff, coef_diff_pl  )
!ESC!    use mod_adm, only: &
!ESC!       ADM_have_pl,  &
!ESC!       ADM_have_sgp, &
!ESC!       ADM_gall_1d,  &
!ESC!       ADM_gmin,     &
!ESC!       ADM_gmax,     &
!ESC!       ADM_gslf_pl,  &
!ESC!       ADM_gmin_pl,  &
!ESC!       ADM_gmax_pl
!ESC!    use mod_gmtr, only: &
!ESC!       P_RAREA => GMTR_p_RAREA, &
!ESC!       T_RAREA => GMTR_t_RAREA, &
!ESC!       HNX     => GMTR_a_HNX,   &
!ESC!       TNX     => GMTR_a_TNX,   &
!ESC!       TN2X    => GMTR_a_TN2X,  &
!ESC!       GMTR_p_nmax,             &
!ESC!       GMTR_t_nmax,             &
!ESC!       GMTR_a_nmax,             &
!ESC!       GMTR_a_nmax_pl
  implicit none

  real(RP), intent(in)  :: GMTR_p      (ADM_gall    ,K0,ADM_lall    ,      GMTR_p_nmax   )
  real(RP), intent(in)  :: GMTR_p_pl   (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_p_nmax   )
  real(RP), intent(in)  :: GMTR_t      (ADM_gall    ,K0,ADM_lall    ,TI:TJ,GMTR_t_nmax   )
  real(RP), intent(in)  :: GMTR_t_pl   (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_t_nmax   )
  real(RP), intent(in)  :: GMTR_a      (ADM_gall    ,K0,ADM_lall    ,AI:AJ,GMTR_a_nmax   )
  real(RP), intent(in)  :: GMTR_a_pl   (ADM_gall_pl,K0,ADM_lall_pl,      GMTR_a_nmax_pl)
  real(RP), intent(out) :: coef_intp   (ADM_gall    ,1:3,ADM_nxyz,TI:TJ,ADM_lall    )
  real(RP), intent(out) :: coef_intp_pl(ADM_gall_pl,1:3,ADM_nxyz,      ADM_lall_pl)
  real(RP), intent(out) :: coef_diff   (ADM_gall,1:6        ,ADM_nxyz,ADM_lall    )
  real(RP), intent(out) :: coef_diff_pl(         1:ADM_vlink,ADM_nxyz,ADM_lall_pl)

  integer  :: gmin, gmax, iall, gall, nxyz, lall, gminm1

  integer  :: ij
  integer  :: ip1j, ijp1
  integer  :: im1j, ijm1, im1jm1

  integer  :: g, l, d, n, v, hn, tn, tn2
  !---------------------------------------------------------------------------

```

Input arguments are the same with the previous subroutines, the metrics calculated by the subroutines described before in this section. But the different from them, output arguments this subroutine calculates are two kinds, `coef_intp`, `coef_diff`, and those for the pole region. `coef_intp` is used to calculate the interpolated value at the vertex points of the control cell from the center point of the cell, and `coef_diff` is used to calculate flux convergence by using the value at the mid-point of the edge of the cell. These coefficients are used not only for the diffusion operatror, but also the operator of horisontal divergence damping. See the source code of subroutine `OPRT_diffusion` in section 2.2.

Main part of this subroutine is consist of three section. The first section is as follows.

```
   !if( IO_L ) write(IO_FID_LOG,*) '*** setup coefficient of diffusion operator'

   gmin = (ADM_gmin-1)*ADM_gall_1d + ADM_gmin
   gmax = (ADM_gmax-1)*ADM_gall_1d + ADM_gmax
   iall = ADM_gall_1d
   gall = ADM_gall
   nxyz = ADM_nxyz
   lall = ADM_lall

   !$omp parallel workshare
   coef_intp   (:,:,:,:,:) = 0.0_RP
   coef_diff   (:,:,:,:)   = 0.0_RP
   !$omp end parallel workshare
   coef_intp_pl(:,:,:,  :) = 0.0_RP
   coef_diff_pl(  :,:,:)   = 0.0_RP

   gminm1 = (ADM_gmin-1-1)*ADM_gall_1d + ADM_gmin-1

   !$omp parallel do default(none),private(g,d,l,tn,ij,ip1j,ijp1), &
   !$omp shared(gminm1,gmax,iall,gall,nxyz,lall,coef_intp,GMTR_t,GMTR_a), &
   !$omp collapse(2)
   do l = 1, lall
   do d = 1, nxyz
      tn = d + TNX - 1

      do g = gminm1, gmax
         ij     = g
         ip1j   = g + 1
         ijp1   = g + iall

         coef_intp(ij,1,d,TI,l) = ( + GMTR_a(ij  ,k0,l,AIJ,tn) - GMTR_a(ij  ,k0,l,AI ,tn) ) &
                                * 0.5_RP * GMTR_t(ij,k0,l,TI,T_RAREA)
         coef_intp(ij,2,d,TI,l) = ( - GMTR_a(ij  ,k0,l,AI ,tn) - GMTR_a(ip1j,k0,l,AJ ,tn) ) &
                                * 0.5_RP * GMTR_t(ij,k0,l,TI,T_RAREA)
         coef_intp(ij,3,d,TI,l) = ( - GMTR_a(ip1j,k0,l,AJ ,tn) + GMTR_a(ij  ,k0,l,AIJ,tn) ) &
                                * 0.5_RP * GMTR_t(ij,k0,l,TI,T_RAREA)

         coef_intp(ij,1,d,TJ,l) = ( + GMTR_a(ij  ,k0,l,AJ ,tn) - GMTR_a(ij  ,k0,l,AIJ,tn) ) &
                                * 0.5_RP * GMTR_t(ij,k0,l,TJ,T_RAREA)
         coef_intp(ij,2,d,TJ,l) = ( - GMTR_a(ij  ,k0,l,AIJ,tn) + GMTR_a(ijp1,k0,l,AI ,tn) ) &
                                * 0.5_RP * GMTR_t(ij,k0,l,TJ,T_RAREA)
         coef_intp(ij,3,d,TJ,l) = ( + GMTR_a(ijp1,k0,l,AI ,tn) + GMTR_a(ij  ,k0,l,AJ ,tn) ) &
                                * 0.5_RP * GMTR_t(ij,k0,l,TJ,T_RAREA)
      enddo
   enddo ! loop d
   enddo ! loop l
   !$omp end parallel do

```

In this section `coef_intp` is calculated.

The second section is as follows.

```
   !$omp parallel default(none),private(g,d,l,hn,ij,im1j,ijm1,im1jm1), &
   !$omp shared(ADM_have_sgp,gmin,gmax,iall,gall,nxyz,lall,coef_diff,GMTR_p,GMTR_a)
   do l = 1, lall
   do d = 1, nxyz
      hn = d + HNX - 1

      !$omp do
      do g = gmin, gmax
         ij     = g
         im1j   = g - 1
         im1jm1 = g - iall - 1
         ijm1   = g - iall

         coef_diff(ij,1,d,l) = + GMTR_a(ij    ,k0,l,AIJ,hn) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
         coef_diff(ij,2,d,l) = + GMTR_a(ij    ,k0,l,AJ ,hn) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
         coef_diff(ij,3,d,l) = - GMTR_a(im1j  ,k0,l,AI ,hn) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
         coef_diff(ij,4,d,l) = - GMTR_a(im1jm1,k0,l,AIJ,hn) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
         coef_diff(ij,5,d,l) = - GMTR_a(ijm1  ,k0,l,AJ ,hn) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
         coef_diff(ij,6,d,l) = + GMTR_a(ij    ,k0,l,AI ,hn) * 0.5_RP * GMTR_p(ij,k0,l,P_RAREA)
      enddo
      !$omp end do

      if ( ADM_have_sgp(l) ) then ! pentagon
         !$omp master
         coef_diff(gmin,5,d,l) = 0.0_RP
         !$omp end master
```

```
      endif
    enddo ! loop d
    enddo ! loop l
    !$omp end parallel

```

In this section `coef_diff` is calculated.

The last section is as follows.

```
    if ( ADM_have_pl ) then
       n  = ADM_gslf_pl

       do l = 1, ADM_lall_pl

          do d = 1, ADM_nxyz
             hn  = d + HNX  - 1
             tn  = d + TNX  - 1
             tn2 = d + TN2X - 1

             do v = ADM_gmin_pl, ADM_gmax_pl
                ij   = v
                ijp1 = v + 1
                if( ijp1 == ADM_gmax_pl+1 ) ijp1 = ADM_gmin_pl

                coef_intp_pl(v,1,d,l) = - GMTR_a_pl(ijp1,k0,l,tn ) + GMTR_a_pl(ij  ,k0,l,tn )
                coef_intp_pl(v,2,d,l) = + GMTR_a_pl(ij  ,k0,l,tn ) + GMTR_a_pl(ij  ,k0,l,tn2)
                coef_intp_pl(v,3,d,l) = + GMTR_a_pl(ij  ,k0,l,tn2) - GMTR_a_pl(ijp1,k0,l,tn )

                coef_intp_pl(v,:,d,l) = coef_intp_pl(v,:,d,l) * 0.5_RP * GMTR_t_pl(v,k0,l,T_RAREA)

                coef_diff_pl(v-1,d,l) = GMTR_a_pl(v,k0,l,hn) * 0.5_RP * GMTR_p_pl(n,k0,l,P_RAREA)
             enddo
          enddo

       enddo ! l loop
    endif

    return
  end subroutine OPRT_diffusion_setup
```

This section is for the pole region.

2.8.3 Input data and result

Max/min/sum of input/output data of the kernel subroutine are output as a log. Below is an example of `$IAB_SYS=Ubuntu-gnu-ompi` case.

```
### Input ###
+check[GRD_x           ] max=  6.3712200000000000E+06,min= -2.8628180616668505E+06,sum=  1.6166957233794562E+11
+check[GRD_x_pl        ] max=  6.3712200000000000E+06,min= -6.3712200000000000E+06,sum= -3.6880373954772949E-07
+check[GRD_xt          ] max=  4.0592444288400000E+13,min= -2.8414384541049926E+06,sum=  6.3080977453379920E+16
+check[GRD_xt_pl       ] max=  6.3712200000000000E+06,min= -6.3712200000000000E+06,sum= -2.6449561119079590E-07
+check[GRD_s           ] max=  1.5707963267948966E+00,min= -1.2566370614358098E+00,sum=  1.9488544754099312E+04
+check[GRD_s_pl        ] max=  3.1415926535897931E+00,min= -2.5132741228746029E+00,sum=  3.1415926535791883E+00
+check[check_GMTR_p    ] max=  3.1937623126102595E+09,min= -1.0000000000000000E+00,sum=  5.2568771941077820E+13
+check[check_GMTR_p_pl ] max=  2.6945919723204341E+09,min= -1.0000000000000000E+00,sum=  3.0761871438935734E+10
+check[check_GMTR_t    ] max=  1.5988904134394393E+09,min=  0.0000000000000000E+00,sum=  5.1787428409617836E+13
+check[check_GMTR_t_pl ] max=  1.9722549492980933E+09,min=  0.0000000000000000E+00,sum=  1.9722549502848648E+10
+check[check_GMTR_a    ] max=  6.3362880879999531E+04,min= -6.0857794015098916E+04,sum=  7.9258961023823655E+08
+check[check_GMTR_a_pl ] max=  5.7873883883004550E+04,min= -5.7872753043726123E+04,sum= -4.3064210331067443E-07
+check[check_OPRT_coef_] max=  7.7272073615122900E-06,min= -7.7272073610225610E-06,sum= -3.8056606893040025E-03
+check[check_OPRT_coef_] max=  6.2151734035229484E-06,min= -6.2403951822043815E-06,sum=  4.0763460586613204E-21
+check[check_OPRT_coef_] max=  8.1249192032094562E-06,min= -7.0395716054136067E-06,sum=  2.3097297195178346E-03
+check[check_OPRT_coef_] max=  6.2404355543912712E-06,min= -6.2404355543546362E-06,sum=  2.0117032497291439E-21
+check[check_OPRT_coef_] max=  7.7272073615122900E-06,min= -1.2825075745577413E-05,sum=  1.8320196095290668E-15
+check[check_OPRT_coef_] max=  6.2151734035229484E-06,min= -6.2403951822043815E-06,sum= -8.3793581340078915E-17
+check[check_OPRT_coef_] max=  3.5452458387465835E-10,min= -1.7726229192139183E-09,sum= -2.7069899136104107E-20
+check[check_OPRT_coef_] max=  6.4022976413393270E-11,min= -3.2011488205582575E-10,sum= -1.0339757656912846E-25
+check[check_OPRT_coef_] max=  3.6115131385825805E-05,min= -3.3754836603619669E-05,sum= -1.5562860020283464E-02
+check[check_OPRT_coef_] max=  1.4672104347081161E-05,min= -1.4672104347198630E-05,sum=  6.2882931910771206E-19
+check[check_OPRT_coef_] max=  8.2996786251499792E-06,min= -8.2996786252745083E-06,sum= -1.9028303446517686E-03
+check[check_OPRT_coef_] max=  8.7267985478094788E-06,min= -8.7267985478605143E-06,sum=  5.4196873734474373E-21
### Output ###
+check[GMTR_p          ] max=  3.1937623126102595E+09,min= -1.0000000000000000E+00,sum=  5.2568771941077820E+13
```

```
 +check[GMTR_p_pl       ] max=  2.6945919723204341E+09,min= -1.0000000000000000E+00,sum=  3.0761871438935734E+10
 +check[GMTR_t          ] max=  1.5988904134394393E+09,min=  0.0000000000000000E+00,sum=  5.1787428409617836E+13
 +check[GMTR_t_pl       ] max=  1.9722549492980933E+09,min=  0.0000000000000000E+00,sum=  1.9722549502848648E+10
 +check[GMTR_a          ] max=  6.3362880879999531E+04,min= -6.0857794015098916E+04,sum=  7.9258961023823655E+08
 +check[GMTR_a_pl       ] max=  5.7873883883004550E+04,min= -5.7872753043726123E+04,sum= -4.3064210331067443E-07
 +check[OPRT_coef_div   ] max=  7.7272073615122900E-06,min= -7.7272073610225610E-06,sum= -3.8056606893040025E-03
 +check[OPRT_coef_div_pl] max=  6.2151734035229484E-06,min= -6.2403951822043815E-06,sum=  4.0763460586613204E-21
 +check[OPRT_coef_rot   ] max=  8.1249192032094562E-06,min= -7.0395716054136067E-06,sum=  2.3097297195178346E-03
 +check[OPRT_coef_rot_pl] max=  6.2404355543912712E-06,min= -6.2404355543546362E-06,sum=  2.0117032497291439E-21
 +check[OPRT_coef_grad  ] max=  7.7272073615122900E-06,min= -1.2825075745577413E-05,sum=  1.8320196095290668E-15
 +check[OPRT_coef_grad_p] max=  6.2151734035229484E-06,min= -6.2403951822043815E-06,sum= -8.3793581340078915E-17
 +check[OPRT_coef_lap   ] max=  3.5452458387465835E-10,min= -1.7726229192139183E-09,sum= -2.7069899136104107E-20
 +check[OPRT_coef_lap_pl] max=  6.4022976413393270E-11,min= -3.2011488205582575E-10,sum= -1.0339757656912846E-25
 +check[OPRT_coef_intp  ] max=  3.6115131385825805E-05,min= -3.3754836603619669E-05,sum= -1.5562860020283464E-02
 +check[OPRT_coef_intp_p] max=  1.4672104347081161E-05,min= -1.4672104347198630E-05,sum=  6.2882931910771206E-19
 +check[OPRT_coef_diff  ] max=  8.2996786251499792E-06,min= -8.2996786252745083E-06,sum= -1.9028303446517686E-03
 +check[OPRT_coef_diff_p] max=  8.7267985478094788E-06,min= -8.7267985478605143E-06,sum=  5.4196873734474373E-21
 ### Validation : point-by-point diff ###
 +check[check_GMTR_p    ] max=  3.3306690738754696E-16,min= -3.3306690738754696E-16,sum=  8.0781261129099502E-15
 +check[check_GMTR_p_pl ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
 +check[check_GMTR_t    ] max=  5.5511151231257827E-17,min= -2.3841857910156250E-07,sum= -2.3841857899054020E-07
 +check[check_GMTR_t_pl ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
 +check[check_GMTR_a    ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
 +check[check_GMTR_a_pl ] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
 +check[check_OPRT_coef_] max=  8.4703294725430034E-22,min= -5.7240898388669515E-22,sum=  2.2830184906463564E-21
 +check[check_OPRT_coef_] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
 +check[check_OPRT_coef_] max=  5.7271917661640254E-22,min= -8.4703294725430034E-22,sum= -9.7783088161424784E-22
 +check[check_OPRT_coef_] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
 +check[check_OPRT_coef_] max=  8.4703294725430034E-22,min= -2.2896359355467806E-21,sum=  4.2351647362715017E-22
 +check[check_OPRT_coef_] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
 +check[check_OPRT_coef_] max=  5.1698788284564230E-26,min=  0.0000000000000000E+00,sum=  7.7548182426846345E-26
 +check[check_OPRT_coef_] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
 +check[check_OPRT_coef_] max=  1.6940658945086007E-21,min= -3.3881317890172014E-21,sum= -2.9646153153900512E-21
 +check[check_OPRT_coef_] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
 +check[check_OPRT_coef_] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
 +check[check_OPRT_coef_] max=  0.0000000000000000E+00,min=  0.0000000000000000E+00,sum=  0.0000000000000000E+00
 *** Finish kernel


Check the lines below \src{``Validation : point-by-point diff''} line,
that shows difference between calculated output array and
pre-calculated reference array.
These should be zero or enough small to be acceptable.

There are sample output log files in \file{reference/}
in each kernel program directory, for reference purpose.




\subsection{Sample of perfomance result}

Here's an example of the performance result part of the log output.
Below is an example executed with the machine environment described in \autoref{s:measuring_env}.
%
Note that in this program kernel part is iterated one time.

\begin{LstLog}
 *** Computational Time Report
 *** ID=001 : MAIN_dyn_metrics                T=     0.108 N=      1
```

2.9 communication

2.9.1 Description

Kernel `communication` is taken from the original subroutine `COMM_setup` in *NICAM*. This subroutine is originally defined in module `mod_comm`. This module defines all communication stuff. Subroutine `COMM_setup` prepares necessary information for communication, such as connection list of regions.

As described in subsection 2.1.6 this kernel only needs MPI library to compile/execute.

2.9.2 Discretization and code

(1) `COMM_setup`

Main subroutine of this kernel program `COMM_setup` is as follows.

```
subroutine COMM_setup
  use mod_process, only: &
     PRC_MPIstop
  use mod_adm, only: &
     RGNMNG_r2p_pl, &
     I_NPL,         &
     I_SPL
  implicit none

  namelist / COMMPARAM / &
     COMM_apply_barrier, &
     COMM_varmax,        &
     debug,              &
     testonly

  integer :: ierr
  !-------------------------------------------------------------------------

  !--- read parameters
  write(IO_FID_LOG,*)
  write(IO_FID_LOG,*) '+++␣Module[comm]/Category[common␣share]'
  rewind(IO_FID_CONF)
  read(IO_FID_CONF,nml=COMMPARAM,iostat=ierr)
  if ( ierr < 0 ) then
     write(IO_FID_LOG,*) '***␣COMMPARAM␣is␣not␣specified.␣use␣default.'
  elseif( ierr > 0 ) then
     write(*         ,*) 'xxx␣Not␣appropriate␣names␣in␣namelist␣COMMPARAM.␣STOP.'
     write(IO_FID_LOG,*) 'xxx␣Not␣appropriate␣names␣in␣namelist␣COMMPARAM.␣STOP.'
     call PRC_MPIstop
  endif
  write(IO_FID_LOG,nml=COMMPARAM)

  if ( RP == DP ) then
     COMM_datatype = MPI_DOUBLE_PRECISION
  elseif( RP == SP ) then
     COMM_datatype = MPI_REAL
  else
     write(*,*) 'xxx␣precision␣is␣not␣supportd'
     call PRC_MPIstop
  endif

  if (       RGNMNG_r2p_pl(I_NPL) < 0 &
       .AND. RGNMNG_r2p_pl(I_SPL) < 0 ) then
     COMM_pl = .false.
  endif

  write(IO_FID_LOG,*)
  write(IO_FID_LOG,*) '======␣communication␣information␣======'

  call COMM_list_generate

  call COMM_sortdest
  call COMM_sortdest_pl
  call COMM_sortdest_singular

  allocate( REQ_list( Recv_nmax_r2r + Send_nmax_r2r &
                    + Recv_nmax_p2r + Send_nmax_p2r &
                    + Recv_nmax_r2p + Send_nmax_r2p ) )

  if( testonly ) call COMM_debugtest

  return
end subroutine COMM_setup
```

This subroutine calls five private subroutines after reading control namelist `COMMPARAM`. Note that as shown below, control variable `testonly` is set as true, `COMM_debugtest` is also called.

(2) `COMM_list_generate`

Subroutine `COMM_list_generate` is to setup an array `rellist`, that contains several information about the relation between grid points including halo.

Initial part of this subroutine is as follows.

```
!-------------------------------------------------------------------------------
!> Generate inner grid -> halo communication list
subroutine COMM_list_generate
  use mod_adm, only: &
     ADM_prc_me,      &
     ADM_lall,        &
     ADM_gall,        &
     ADM_gmax,        &
     ADM_gmin,        &
     RGNMNG_r2lp,     &
     RGNMNG_l2r,      &
     RGNMNG_edge_tab, &
     RGNMNG_vert_num, &
     RGNMNG_vert_tab, &
     I_prc,           &
     I_RGNID,         &
     I_DIR,           &
     I_SW,            &
     I_NW,            &
     I_NE,            &
     I_SE,            &
     I_W,             &
     I_N,             &
     I_E,             &
     I_S
  implicit none

  integer :: ginnar

  integer :: prc, prc_rmt
  integer :: rgnid, rgnid_rmt
  integer :: i, j, i_rmt, j_rmt

  integer :: n, l, cnt
  !-----------------------------------------------------------------------------

  ginnar = ADM_gmax - ADM_gmin + 1

  allocate( rellist(rellist_vindex,ADM_gall*ADM_lall) )

```

This subroutine is private in module, there are no arguments, but using `use` to include from other module. Variables with prefix `ADM_` define problem size, and variables with prefix `RGNMNG_` define tables to manage the relation between regions and processes. Variables with prefix `I_` are the constant index and are used to specify the kind of quantity. For example, `RGNMNG_r2lp(I_prc,rgnid)` means the process number that manage given region with `rgnid`

The main part of this subroutine is a pretty long l-loop, that is consist of two sections. The first section is as follows.

```
  cnt = 0
  do l = 1, ADM_lall
     rgnid = RGNMNG_l2r(l)
     prc   = ADM_prc_me

     !---< South West >---

     ! NE -> SW halo
     if ( RGNMNG_edge_tab(I_DIR,I_SW,rgnid) == I_NE ) then
        rgnid_rmt = RGNMNG_edge_tab(I_RGNID,I_SW,rgnid)
        prc_rmt   = RGNMNG_r2lp(I_prc,rgnid_rmt)

        do n = 1, ginnar
           cnt = cnt + 1

           i     = ADM_gmin - 1 + n
           j     = ADM_gmin - 1
           i_rmt = ADM_gmin - 1 + n
           j_rmt = ADM_gmax
```

```
            rellist(I_recv_grid,cnt) = suf(i,j)
            rellist(I_recv_rgn, cnt) = rgnid
            rellist(I_recv_prc, cnt) = prc
            rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
            rellist(I_send_rgn, cnt) = rgnid_rmt
            rellist(I_send_prc, cnt) = prc_rmt
         enddo
      endif

      ! SE -> SW halo (Southern Hemisphere, Edge of diamond)
      if ( RGNMNG_edge_tab(I_DIR,I_SW,rgnid) == I_SE ) then
         rgnid_rmt = RGNMNG_edge_tab(I_RGNID,I_SW,rgnid)
         prc_rmt   = RGNMNG_r2lp(I_prc,rgnid_rmt)

         do n = 1, ginnar
            cnt = cnt + 1

            i     = ADM_gmin - 1 + n
            j     = ADM_gmin - 1
            i_rmt = ADM_gmax
            j_rmt = ADM_gmax + 1 - n ! reverse order

            rellist(I_recv_grid,cnt) = suf(i,j)
            rellist(I_recv_rgn, cnt) = rgnid
            rellist(I_recv_prc, cnt) = prc
            rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
            rellist(I_send_rgn, cnt) = rgnid_rmt
            rellist(I_send_prc, cnt) = prc_rmt
         enddo
      endif

      !---< North West >---

      ! SE -> NW
      if ( RGNMNG_edge_tab(I_DIR,I_NW,rgnid) == I_SE ) then
         rgnid_rmt = RGNMNG_edge_tab(I_RGNID,I_NW,rgnid)
         prc_rmt   = RGNMNG_r2lp(I_prc,rgnid_rmt)

         do n = 1, ginnar
            cnt = cnt + 1

            i     = ADM_gmin - 1
            j     = ADM_gmin - 1 + n
            i_rmt = ADM_gmax
            j_rmt = ADM_gmin - 1 + n

            rellist(I_recv_grid,cnt) = suf(i,j)
            rellist(I_recv_rgn, cnt) = rgnid
            rellist(I_recv_prc, cnt) = prc
            rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
            rellist(I_send_rgn, cnt) = rgnid_rmt
            rellist(I_send_prc, cnt) = prc_rmt
         enddo
      endif

      ! NE -> NW  (Northern Hemisphere, Edge of diamond)
      if ( RGNMNG_edge_tab(I_DIR,I_NW,rgnid) == I_NE ) then
         rgnid_rmt = RGNMNG_edge_tab(I_RGNID,I_NW,rgnid)
         prc_rmt   = RGNMNG_r2lp(I_prc,rgnid_rmt)

         do n = 1, ginnar
            cnt = cnt + 1

            i     = ADM_gmin - 1
            j     = ADM_gmin - 1 + n
            i_rmt = ADM_gmax + 1 - n ! reverse order
            j_rmt = ADM_gmax

            rellist(I_recv_grid,cnt) = suf(i,j)
            rellist(I_recv_rgn, cnt) = rgnid
            rellist(I_recv_prc, cnt) = prc
            rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
            rellist(I_send_rgn, cnt) = rgnid_rmt
            rellist(I_send_prc, cnt) = prc_rmt
         enddo
      endif

      !---< North East >---

      ! SW -> NE
      if ( RGNMNG_edge_tab(I_DIR,I_NE,rgnid) == I_SW ) then
```

```
         rgnid_rmt = RGNMNG_edge_tab(I_RGNID,I_NE,rgnid)
         prc_rmt   = RGNMNG_r2lp(I_prc,rgnid_rmt)

         do n = 1, ginnar
            cnt = cnt + 1

            i     = ADM_gmin - 1 + n
            j     = ADM_gmax + 1
            i_rmt = ADM_gmin - 1 + n
            j_rmt = ADM_gmin

            rellist(I_recv_grid,cnt) = suf(i,j)
            rellist(I_recv_rgn, cnt) = rgnid
            rellist(I_recv_prc, cnt) = prc
            rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
            rellist(I_send_rgn, cnt) = rgnid_rmt
            rellist(I_send_prc, cnt) = prc_rmt
         enddo
      endif

      ! NW -> NE  (Northern Hemisphere, Edge of diamond)
      if ( RGNMNG_edge_tab(I_DIR,I_NE,rgnid) == I_NW ) then
         rgnid_rmt = RGNMNG_edge_tab(I_RGNID,I_NE,rgnid)
         prc_rmt   = RGNMNG_r2lp(I_prc,rgnid_rmt)

         do n = 1, ginnar
            cnt = cnt + 1

            i     = ADM_gmin     + n ! shift 1 grid
            j     = ADM_gmax + 1
            i_rmt = ADM_gmin
            j_rmt = ADM_gmax + 1 - n ! reverse order

            rellist(I_recv_grid,cnt) = suf(i,j)
            rellist(I_recv_rgn, cnt) = rgnid
            rellist(I_recv_prc, cnt) = prc
            rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
            rellist(I_send_rgn, cnt) = rgnid_rmt
            rellist(I_send_prc, cnt) = prc_rmt
         enddo
      endif

      !---< South East >---

      ! NW -> SE
      if ( RGNMNG_edge_tab(I_DIR,I_SE,rgnid) == I_NW ) then
         rgnid_rmt = RGNMNG_edge_tab(I_RGNID,I_SE,rgnid)
         prc_rmt   = RGNMNG_r2lp(I_prc,rgnid_rmt)

         do n = 1, ginnar
            cnt = cnt + 1

            i     = ADM_gmax + 1
            j     = ADM_gmin - 1 + n
            i_rmt = ADM_gmin
            j_rmt = ADM_gmin - 1 + n

            rellist(I_recv_grid,cnt) = suf(i,j)
            rellist(I_recv_rgn, cnt) = rgnid
            rellist(I_recv_prc, cnt) = prc
            rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
            rellist(I_send_rgn, cnt) = rgnid_rmt
            rellist(I_send_prc, cnt) = prc_rmt
         enddo
      endif

      ! SW -> SE  (Southern Hemisphere, Edge of diamond)
      if ( RGNMNG_edge_tab(I_DIR,I_SE,rgnid) == I_SW ) then
         rgnid_rmt = RGNMNG_edge_tab(I_RGNID,I_SE,rgnid)
         prc_rmt   = RGNMNG_r2lp(I_prc,rgnid_rmt)

         do n = 1, ginnar
            cnt = cnt + 1

            i     = ADM_gmax + 1
            j     = ADM_gmin     + n ! shift 1 grid
            i_rmt = ADM_gmax + 1 - n ! reverse order
            j_rmt = ADM_gmin

            rellist(I_recv_grid,cnt) = suf(i,j)
            rellist(I_recv_rgn, cnt) = rgnid
            rellist(I_recv_prc, cnt) = prc
```

```
224            rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
225            rellist(I_send_rgn, cnt) = rgnid_rmt
226            rellist(I_send_prc, cnt) = prc_rmt
227         enddo
228      endif
229
```

This section sets `rellist` at four edges of each region. The first dimension of `rellist` has the size 6, they specify the pair of source and destination grid point. The data is transferred from source grid to destination grid in halo communication. The second dimension is for the serial number of the grid. That is, `rellist` manages 6 kind of quantities for each inner grid, first three are one-dimensioned grid index from (i, j), region number and process number, respectively. The grid points identified by these three indices are the destination of data transfer. The latter three are the same but of the informations of the source grid points.

Remember that the shape of the region in *NICAM* is a diamond, and setting of `rellist` is separated for each edge of a diamond. For example, `( RGNMNG_edge_tab(I_DIR,I_SW,rgnid) == I_NE )`(l.49) means that whether the south-west edge of the region with `rgnid` is linked with the north-east edge of neighbouring region. Therefore, this `IF`-clause sets `rellist` as that the halo grid points of this region are linked with the corresponding inner grid points of the region `rgnid_rmt` which is managed by the process `prc_rmt`.

The continuing section is as follows.

```
230      !---< Vertex : link to the next next region >---
231
232      ! West Vertex
233      if ( RGNMNG_vert_num(I_W,rgnid) == 4 ) then
234         rgnid_rmt = RGNMNG_vert_tab(I_RGNID,I_W,rgnid,3)
235         prc_rmt   = RGNMNG_r2lp(I_prc,rgnid_rmt)
236
237         cnt = cnt + 1
238
239         i = ADM_gmin - 1
240         j = ADM_gmin - 1
241
242         if ( RGNMNG_vert_tab(I_DIR,I_W,rgnid,3) == I_N ) then
243            i_rmt = ADM_gmin
244            j_rmt = ADM_gmax
245         elseif( RGNMNG_vert_tab(I_DIR,I_W,rgnid,3) == I_E ) then
246            i_rmt = ADM_gmax
247            j_rmt = ADM_gmax
248         elseif( RGNMNG_vert_tab(I_DIR,I_W,rgnid,3) == I_S ) then
249            i_rmt = ADM_gmax
250            j_rmt = ADM_gmin
251         endif
252
253         rellist(I_recv_grid,cnt) = suf(i,j)
254         rellist(I_recv_rgn, cnt) = rgnid
255         rellist(I_recv_prc, cnt) = prc
256         rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
257         rellist(I_send_rgn, cnt) = rgnid_rmt
258         rellist(I_send_prc, cnt) = prc_rmt
259      endif
260
261      ! North Vertex
262      if ( RGNMNG_vert_num(I_N,rgnid) == 4 ) then
263         rgnid_rmt = RGNMNG_vert_tab(I_RGNID,I_N,rgnid,3)
264         prc_rmt   = RGNMNG_r2lp(I_prc,rgnid_rmt)
265
266         ! known as north pole point
267         if ( RGNMNG_vert_tab(I_DIR,I_N,rgnid,3) == I_W ) then
268            cnt = cnt + 1
269
270            i     = ADM_gmin
271            j     = ADM_gmax + 1
272            i_rmt = ADM_gmin
273            j_rmt = ADM_gmin
274
275            rellist(I_recv_grid,cnt) = suf(i,j)
276            rellist(I_recv_rgn, cnt) = rgnid
277            rellist(I_recv_prc, cnt) = prc
278            rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
279            rellist(I_send_rgn, cnt) = rgnid_rmt
280            rellist(I_send_prc, cnt) = prc_rmt
281         endif
282
```

```
            ! unused vertex point
            cnt = cnt + 1

            i = ADM_gmin - 1
            j = ADM_gmax + 1

            if ( RGNMNG_vert_tab(I_DIR,I_N,rgnid,3) == I_W ) then
               i_rmt = ADM_gmin
               j_rmt = ADM_gmin + 1
            elseif( RGNMNG_vert_tab(I_DIR,I_N,rgnid,3) == I_N ) then
               i_rmt = ADM_gmin
               j_rmt = ADM_gmax
            elseif( RGNMNG_vert_tab(I_DIR,I_N,rgnid,3) == I_E ) then
               i_rmt = ADM_gmax
               j_rmt = ADM_gmax
            elseif( RGNMNG_vert_tab(I_DIR,I_N,rgnid,3) == I_S ) then
               i_rmt = ADM_gmax
               j_rmt = ADM_gmin
            endif

            rellist(I_recv_grid,cnt) = suf(i,j)
            rellist(I_recv_rgn, cnt) = rgnid
            rellist(I_recv_prc, cnt) = prc
            rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
            rellist(I_send_rgn, cnt) = rgnid_rmt
            rellist(I_send_prc, cnt) = prc_rmt
         endif

         ! East Vertex
         if ( RGNMNG_vert_num(I_E,rgnid) == 4 ) then
            if ( RGNMNG_vert_tab(I_DIR,I_E,rgnid,3) == I_W ) then
               rgnid_rmt = RGNMNG_vert_tab(I_RGNID,I_E,rgnid,3)
               prc_rmt   = RGNMNG_r2lp(I_prc,rgnid_rmt)

               cnt = cnt + 1

               i     = ADM_gmax + 1
               j     = ADM_gmax + 1
               i_rmt = ADM_gmin
               j_rmt = ADM_gmin

               rellist(I_recv_grid,cnt) = suf(i,j)
               rellist(I_recv_rgn, cnt) = rgnid
               rellist(I_recv_prc, cnt) = prc
               rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
               rellist(I_send_rgn, cnt) = rgnid_rmt
               rellist(I_send_prc, cnt) = prc_rmt
            endif
         endif

         ! South Vertex
         if ( RGNMNG_vert_num(I_S,rgnid) == 4 ) then
            rgnid_rmt = RGNMNG_vert_tab(I_RGNID,I_S,rgnid,3)
            prc_rmt   = RGNMNG_r2lp(I_prc,rgnid_rmt)

            ! known as south pole point
            if ( RGNMNG_vert_tab(I_DIR,I_S,rgnid,3) == I_W ) then
               cnt = cnt + 1

               i     = ADM_gmax + 1
               j     = ADM_gmin
               i_rmt = ADM_gmin
               j_rmt = ADM_gmin

               rellist(I_recv_grid,cnt) = suf(i,j)
               rellist(I_recv_rgn, cnt) = rgnid
               rellist(I_recv_prc, cnt) = prc
               rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
               rellist(I_send_rgn, cnt) = rgnid_rmt
               rellist(I_send_prc, cnt) = prc_rmt
            endif

            ! unused vertex point
            cnt = cnt + 1

            i = ADM_gmax + 1
            j = ADM_gmin - 1

            if ( RGNMNG_vert_tab(I_DIR,I_S,rgnid,3) == I_W ) then
               i_rmt = ADM_gmin + 1
               j_rmt = ADM_gmin
            elseif( RGNMNG_vert_tab(I_DIR,I_S,rgnid,3) == I_N ) then
```

```
365              i_rmt = ADM_gmin
366              j_rmt = ADM_gmax
367           elseif( RGNMNG_vert_tab(I_DIR,I_S,rgnid,3) == I_E ) then
368              i_rmt = ADM_gmax
369              j_rmt = ADM_gmax
370           elseif( RGNMNG_vert_tab(I_DIR,I_S,rgnid,3) == I_S ) then
371              i_rmt = ADM_gmax
372              j_rmt = ADM_gmin
373           endif
374
375           rellist(I_recv_grid,cnt) = suf(i,j)
376           rellist(I_recv_rgn, cnt) = rgnid
377           rellist(I_recv_prc, cnt) = prc
378           rellist(I_send_grid,cnt) = suf(i_rmt,j_rmt)
379           rellist(I_send_rgn, cnt) = rgnid_rmt
380           rellist(I_send_prc, cnt) = prc_rmt
381        endif
382     enddo ! loop l
383
384     rellist_nmax = cnt
385
386     write(IO_FID_LOG,*)
387     write(IO_FID_LOG,*) "***␣rellist_nmax:", rellist_nmax
388
389     if ( debug ) then
390        write(IO_FID_LOG,*) "---␣Relation␣Table"
391        write(IO_FID_LOG,'(7(A10))') 'Count', '|recv_grid', '|␣recv_rgn', '|␣recv_prc', &
392                                                '|send_grid', '|␣send_rgn', '|␣send_prc'
393        do cnt = 1, rellist_nmax
394           write(IO_FID_LOG,'(7(I10))') cnt, rellist(:,cnt)
395        enddo
396     endif
397
398     return
399   end subroutine COMM_list_generate
```

Similar to the previous secion, this section sets `rellist` additional source-destination relationships of each region. Around the vertices, the data can be transferred from next-next region. In the `IF`-clause at l.233, `( RGNMNG_vert_num(I_W,rgnid) == 4 )` means whether the number of regions around the western vertex of the region with `rgnid` is four, i.e. this vertex is not a singular point, that has only three region links. And `( RGNMNG_vert_tab(I_DIR,I_W,rgnid,3) == I_N )`(l.242) means whether the northern vertex of the next-next region is connected to the western vertex of current region. The regions around the vertex are searched clockwise. Other two `IF`-clauses(l.245 and l.248) are the same meaning but for the east or west vertex of the next-next region.

At the northern and southern vertices, there are the cases applying special treatment. If the region does not contain north pole at the halo, and the north-western edge is connected to the north-eastern edge of neighboring region, We should take one grid from the next-next region into account. Additionally, in northern and southern vertex section, northern-most and southern-most vertex in each region is not used, respectively (See Figure 1.8 in section 1.8). These points are filled by the value of the grid in the self region.

After the l-loop, contents of `rellist` is written to the log (standard output). See subsection 2.9.3 for example.

Also note that even if one region is managed by a single process, all process has the same `rellist` that contains the information of *ALL* regions.

(3) COMM_sortdest

Subroutine `COMM_sortdest` is to sort the region to region connection info.

The first part is as follows.

```
!-------------------------------------------------------------------------------
!> Sort data destination for region <-> region
subroutine COMM_sortdest
  use mod_process, only: &
     PRC_LOCAL_COMM_WORLD, &
     PRC_nprocs
  use mod_adm, only: &
     ADM_prc_me,     &
     ADM_kall,       &
     ADM_gall_1d,    &
```

```fortran
      RGNMNG_lp2r,    &
      RGNMNG_r2lp,    &
      I_l
  implicit none

  integer :: sendbuf1(1)
  integer :: recvbuf1(1)

  integer, allocatable :: sendbuf_info(:)
  integer, allocatable :: recvbuf_info(:)
  integer, allocatable :: sendbuf_list(:,:,:)
  integer, allocatable :: recvbuf_list(:,:,:)
  integer, allocatable :: REQ_list_r2r(:)

  integer :: Recv_nglobal_r2r
  integer :: Send_size_nglobal

  integer :: cnt, irank, ipos
  integer :: totalsize, rank, tag

  integer :: i_from, j_from, r_from, g_from, l_from, p_from
  integer :: i_to, j_to, r_to, g_to, l_to, p_to

  integer :: ierr
  integer :: n, p
  !---------------------------------------------------------------------------

  allocate( Copy_info_r2r(info_vindex) )
  allocate( Recv_info_r2r(info_vindex,Recv_nlim) )
  allocate( Send_info_r2r(info_vindex,Send_nlim) )
  Copy_info_r2r(:)        = -1
  Recv_info_r2r(:,:)      = -1
  Send_info_r2r(:,:)      = -1
  Copy_info_r2r(I_size)   = 0
  Recv_info_r2r(I_size,:) = 0
  Send_info_r2r(I_size,:) = 0

  allocate( Copy_list_r2r(list_vindex,rellist_nmax) )
  allocate( Recv_list_r2r(list_vindex,rellist_nmax,Recv_nlim) )
  allocate( Send_list_r2r(list_vindex,rellist_nmax,Send_nlim) )
  Copy_list_r2r(:,:)   = -1
  Recv_list_r2r(:,:,:) = -1
  Send_list_r2r(:,:,:) = -1

```

This subroutine is also private in this module, there are no arguments. Variables with the prefix `sendbuf` and `recvbuf` are temporal containers used for MPI communication. `Send_info_r2r` and `Recv_info_r2r` specify the number of transferring grid points between two precesses, for the sending side and receiving side, respectively. `Copy_info_r2r` is the number of grid points to copy within the same process. `Copy_info_r2r` is one-dimensional and the size is `info_vindex` that is 3, and the index takes one of `I_size`, `I_prc_from` or `I_prc_to`. `Copy_info_r2r(I_size)` is the number of halo-grid points that exchange with the grid points in other region by memory-copy, and `Copy_info_r2r(I_prc_from)` and `Copy_info_r2r(i_prc_to)` are the process number that the process this region exchange from/to, of course this value is the same with `ADM_prc_me`, the process this region is managed.

On the other hand, `Recv_info_r2r` and `Send_info_r2r` are two-dimensional, the first dimension of them are the same with `Copy_info_r2r`. The size of the second dimension of these are `Recv_nlim` and `Send_nlim`, respectively, they are both 10, and this dimension specifies that which process this process receive from/send to.

`Copy_list_r2r` is two-dimensional, the size of these dimension are `list_vindex`, which is 4, and `rellist_nmax`, which is the number of grid points `rellist` contains. First dimension takes one of `I_grid_from`, `I_l_from` `I_grid_to` or `I_l_to`, they specify the grid point and the (local) region number the value comes from, the grid point and the (local) region number the value goes to, respectively. And second dimension specifies the grid point in this process whose value is exchanged by the memory-copy.

`Recv_list_r2r` and `Send_list_r2r` are similar, but have another dimension, whose size are `Recv_nlim` and `Send_nlim`, respectively. This last dimension has the same meaning with the last dimension of `Recv_info_r2r` and `Send_info_r2r`.

These arrays are set in the next section, as follows.

```fortran
  ! sorting list according to destination
```

```
   do cnt = 1, rellist_nmax

      if ( rellist(I_recv_prc,cnt) == rellist(I_send_prc,cnt) ) then ! no communication
         Copy_info_r2r(I_size) = Copy_info_r2r(I_size) + 1
         ipos                  = Copy_info_r2r(I_size)

         Copy_list_r2r(I_grid_from,ipos) = rellist(I_send_grid,cnt)
         Copy_list_r2r(I_l_from   ,ipos) = RGNMNG_r2lp(I_l,rellist(I_send_rgn,cnt))
         Copy_list_r2r(I_grid_to  ,ipos) = rellist(I_recv_grid,cnt)
         Copy_list_r2r(I_l_to     ,ipos) = RGNMNG_r2lp(I_l,rellist(I_recv_rgn,cnt))
      else ! node-to-node communication
         !--- search existing rank id (identify by prc_from)
         irank = -1
         do n = 1, Recv_nmax_r2r
            if ( Recv_info_r2r(I_prc_from,n) == rellist(I_send_prc,cnt) ) then
               irank = n
               exit
            endif
         enddo

         if ( irank < 0 ) then ! register new rank id
            Recv_nmax_r2r = Recv_nmax_r2r + 1
            irank         = Recv_nmax_r2r

            Recv_info_r2r(I_prc_from,irank) = rellist(I_send_prc,cnt)
            Recv_info_r2r(I_prc_to  ,irank) = rellist(I_recv_prc,cnt)
         endif

         Recv_info_r2r(I_size,irank) = Recv_info_r2r(I_size,irank) + 1
         ipos                        = Recv_info_r2r(I_size,irank)

         Recv_list_r2r(I_grid_from,ipos,irank) = rellist(I_send_grid,cnt)
         Recv_list_r2r(I_l_from   ,ipos,irank) = RGNMNG_r2lp(I_l,rellist(I_send_rgn,cnt))
         Recv_list_r2r(I_grid_to  ,ipos,irank) = rellist(I_recv_grid,cnt)
         Recv_list_r2r(I_l_to     ,ipos,irank) = RGNMNG_r2lp(I_l,rellist(I_recv_rgn,cnt))
      endif
   enddo

   if ( Copy_info_r2r(I_size) > 0 ) then
      Copy_nmax_r2r = 1
      Copy_info_r2r(I_prc_from) = ADM_prc_me
      Copy_info_r2r(I_prc_to  ) = ADM_prc_me
   endif
```

In the next section there are some MPI communication, as follows.

```
   ! get maximum number of rank to communication
   sendbuf1(1) = Recv_nmax_r2r

   call MPI_Allreduce( sendbuf1(1),          & ! [IN]
                       recvbuf1(1),          & ! [OUT]
                       1,                    & ! [IN]
                       MPI_INTEGER,          & ! [IN]
                       MPI_MAX,              & ! [IN]
                       PRC_LOCAL_COMM_WORLD, & ! [IN]
                       ierr                  ) ! [OUT]

   Recv_nglobal_r2r = recvbuf1(1)

   ! Distribute receive request from each rank to all
   allocate( sendbuf_info(info_vindex*Recv_nglobal_r2r) )
   allocate( recvbuf_info(info_vindex*Recv_nglobal_r2r*PRC_nprocs) )
   sendbuf_info(:) = -1

   do irank = 1, Recv_nmax_r2r
      n = (irank-1) * info_vindex

      sendbuf_info(n+I_size    ) = Recv_info_r2r(I_size    ,irank)
      sendbuf_info(n+I_prc_from) = Recv_info_r2r(I_prc_from,irank)
      sendbuf_info(n+I_prc_to  ) = Recv_info_r2r(I_prc_to  ,irank)
   enddo

   totalsize = info_vindex * Recv_nglobal_r2r

   call MPI_Allgather( sendbuf_info(1),      & ! [IN]
                       totalsize,            & ! [IN]
                       MPI_INTEGER,          & ! [IN]
```

```
                        recvbuf_info(1),      & ! [OUT]
                        totalsize,            & ! [IN]
                        MPI_INTEGER,          & ! [IN]
                        PRC_LOCAL_COMM_WORLD, & ! [IN]
                        ierr                  ) ! [OUT]

    Send_size_nglobal = 0

    ! Accept receive request to my rank
    do p  = 1, Recv_nglobal_r2r*PRC_nprocs
       n = (p-1) * info_vindex

       if ( recvbuf_info(n+I_prc_from) == ADM_prc_me ) then
          Send_nmax_r2r = Send_nmax_r2r + 1
          irank         = Send_nmax_r2r

          Send_info_r2r(I_size    ,irank) = recvbuf_info(n+I_size    )
          Send_info_r2r(I_prc_from,irank) = recvbuf_info(n+I_prc_from)
          Send_info_r2r(I_prc_to  ,irank) = recvbuf_info(n+I_prc_to  )
       endif

       Send_size_nglobal = max( Send_size_nglobal, recvbuf_info(n+I_size) )
    enddo

    write(IO_FID_LOG,*)
    write(IO_FID_LOG,*) '*** Recv_nmax_r2r(global) = ', Recv_nglobal_r2r
    write(IO_FID_LOG,*) '*** Recv_nmax_r2r(local)  = ', Recv_nmax_r2r
    write(IO_FID_LOG,*) '*** Send_nmax_r2r(local)  = ', Send_nmax_r2r
    write(IO_FID_LOG,*) '*** Send_size_r2r(global) = ', Send_size_nglobal
    write(IO_FID_LOG,*)
    write(IO_FID_LOG,'(A)')             "|---------------------------------------"
    write(IO_FID_LOG,'(A)')             "|               size  prc_from    prc_to"
    write(IO_FID_LOG,'(A10,3(I10))')    "| Copy_r2r", Copy_info_r2r(:)
    do irank = 1, Recv_nmax_r2r
       write(IO_FID_LOG,'(A10,3(I10))') "| Recv_r2r", Recv_info_r2r(:,irank)
    enddo
    do irank = 1, Send_nmax_r2r
       write(IO_FID_LOG,'(A10,3(I10))') "| Send_r2r", Send_info_r2r(:,irank)
    enddo

```

In this section, shown as several write statements, `Recv_nglobal_r2r`, `Recv_nmax_r2r`, `Send_nmax_r2r` and `Send_size_nglobal` are calculated. `Recv_nglobal_r2r` is the maximum of `Recv_nmax_r2r` in all processes, number of processes to communicate. This value is calculated by `MPI_Allreduce` with `MPI_MAX` operation. Then contents of `Recv_info_r2r` are `MPI_Allgathered` and the contents of `Recv_info_r2r` which related to the current process are additionally stored to the `Send_info_r2r`.

The next section is as follows.

```
    ! Communicate detailed information in each pair
    allocate( REQ_list_r2r(Recv_nmax_r2r+Send_nmax_r2r)  )

    allocate( sendbuf_list(list_vindex,Send_size_nglobal,Recv_nmax_r2r) )
    allocate( recvbuf_list(list_vindex,Send_size_nglobal,Send_nmax_r2r) )
    sendbuf_list(:,:,:) = -1

    REQ_count = 0

    do irank = 1, Send_nmax_r2r
       REQ_count = REQ_count + 1
       totalsize = Send_info_r2r(I_size,irank) * list_vindex
       rank      = Send_info_r2r(I_prc_to  ,irank) - 1 ! rank = prc - 1
       tag       = Send_info_r2r(I_prc_from,irank) - 1 ! rank = prc - 1

       call MPI_IRECV( recvbuf_list(1,1,irank), & ! [OUT]
                       totalsize,               & ! [IN]
                       MPI_INTEGER,             & ! [IN]
                       rank,                    & ! [IN]
                       tag,                     & ! [IN]
                       PRC_LOCAL_COMM_WORLD,    & ! [IN]
                       REQ_list_r2r(REQ_count), & ! [OUT]
                       ierr                     ) ! [OUT]
    enddo

    do irank = 1, Recv_nmax_r2r
       do ipos = 1, Recv_info_r2r(I_size,irank)
          sendbuf_list(:,ipos,irank) = Recv_list_r2r(:,ipos,irank)
       enddo
```

```
      REQ_count = REQ_count + 1
      totalsize = Recv_info_r2r(I_size,irank) * list_vindex
      rank      = Recv_info_r2r(I_prc_from,irank) - 1 ! rank = prc - 1
      tag       = Recv_info_r2r(I_prc_from,irank) - 1 ! rank = prc - 1

      call MPI_ISEND( sendbuf_list(1,1,irank), & ! [IN]
                      totalsize,               & ! [IN]
                      MPI_INTEGER,             & ! [IN]
                      rank,                    & ! [IN]
                      tag,                     & ! [IN]
                      PRC_LOCAL_COMM_WORLD,    & ! [IN]
                      REQ_list_r2r(REQ_count), & ! [OUT]
                      ierr                     ) ! [OUT]
   enddo

   !--- wait packets
   call MPI_WAITALL( Recv_nmax_r2r+Send_nmax_r2r, & ! [IN]
                     REQ_list_r2r(:),             & ! [IN]
                     MPI_STATUSES_IGNORE,         & ! [OUT]
                     ierr                         ) ! [OUT]

   do irank = 1, Send_nmax_r2r
      do ipos = 1, Send_info_r2r(I_size,irank)
         Send_list_r2r(:,ipos,irank) = recvbuf_list(:,ipos,irank)
      enddo
   enddo

```

In this section each process send the contents of `Recv_info_r2r` to the corresponding process, then receive and store them to the corresponding location of `Send_info_r2r` and `Send_list_r2r`. Note that this procedure is a kind of all to all communication, implemented asynchronous communication, `MPI_IRECV`, `MPI_ISEND` and `MPI_WAITALL`.

Final section of this subroutine is as follows.

```
   if ( debug ) then
      write(IO_FID_LOG,*)
      write(IO_FID_LOG,*) "---␣Copy_list_r2r"
      write(IO_FID_LOG,*)
      write(IO_FID_LOG,'(13(A6))') "number", &
                                         "|ifrom","|jfrom","|rfrom","|gfrom","|lfrom","|pfrom", &
                                         "|␣␣ito","|␣␣jto","|␣␣rto","|␣␣gto","|␣␣lto","|␣␣pto"
      do ipos = 1, Copy_info_r2r(I_size)
         g_from = Copy_list_r2r(I_grid_from,ipos)
         l_from = Copy_list_r2r(I_l_from,ipos)
         p_from = Copy_info_r2r(I_prc_from)
         i_from = mod(g_from-1,ADM_gall_1d) + 1
         j_from = (g_from-i_from) / ADM_gall_1d + 1
         r_from = RGNMNG_lp2r(l_from,p_from)
         g_to   = Copy_list_r2r(I_grid_to,ipos)
         l_to   = Copy_list_r2r(I_l_to,ipos)
         p_to   = Copy_info_r2r(I_prc_to)
         i_to   = mod(g_to-1,ADM_gall_1d) + 1
         j_to   = (g_to-i_to) / ADM_gall_1d + 1
         r_to   = RGNMNG_lp2r(l_to,p_to)

         write(IO_FID_LOG,'(13(I6))') ipos, i_from, j_from, r_from, g_from, l_from, p_from, &
                                            i_to  , j_to  , r_to  , g_to  , l_to  , p_to
      enddo

      write(IO_FID_LOG,*)
      write(IO_FID_LOG,*) "---␣Recv_list_r2r"
      do irank = 1, Recv_nmax_r2r
         write(IO_FID_LOG,'(13(A6))') "number", &
                                            "|ifrom","|jfrom","|rfrom","|gfrom","|lfrom","|pfrom", &
                                            "|␣␣ito","|␣␣jto","|␣␣rto","|␣␣gto","|␣␣lto","|␣␣pto"
         do ipos = 1, Recv_info_r2r(I_size,irank)
            g_from = Recv_list_r2r(I_grid_from,ipos,irank)
            l_from = Recv_list_r2r(I_l_from,ipos,irank)
            p_from = Recv_info_r2r(I_prc_from,irank)
            i_from = mod(g_from-1,ADM_gall_1d) + 1
            j_from = (g_from-i_from) / ADM_gall_1d + 1
            r_from = RGNMNG_lp2r(l_from,p_from)
            g_to   = Recv_list_r2r(I_grid_to,ipos,irank)
            l_to   = Recv_list_r2r(I_l_to,ipos,irank)
            p_to   = Recv_info_r2r(I_prc_to,irank)
            i_to   = mod(g_to-1,ADM_gall_1d) + 1
            j_to   = (g_to-i_to) / ADM_gall_1d + 1
```

```
          r_to   = RGNMNG_lp2r(l_to,p_to)

          write(IO_FID_LOG,'(13(I6))') ipos, i_from, j_from, r_from, g_from, l_from, p_from, &
                                             i_to  , j_to  , r_to  , g_to  , l_to  , p_to
       enddo
    enddo

    write(IO_FID_LOG,*)
    write(IO_FID_LOG,*) "---␣Send_list_r2r"
    do irank = 1, Send_nmax_r2r
       write(IO_FID_LOG,'(13(A6))') "number", &
                                      "|ifrom","|jfrom","|rfrom","|gfrom","|lfrom","|pfrom", &
                                      "|␣␣ito","|␣␣jto","|␣␣rto","|␣␣gto","|␣␣lto","|␣␣pto"
       do ipos = 1, Send_info_r2r(I_size,irank)
          g_from = Send_list_r2r(I_grid_from,ipos,irank)
          l_from = Send_list_r2r(I_l_from,ipos,irank)
          p_from = Send_info_r2r(I_prc_from,irank)
          i_from = mod(g_from-1,ADM_gall_1d) + 1
          j_from = (g_from-i_from) / ADM_gall_1d + 1
          r_from = RGNMNG_lp2r(l_from,p_from)
          g_to   = Send_list_r2r(I_grid_to,ipos,irank)
          l_to   = Send_list_r2r(I_l_to,ipos,irank)
          p_to   = Send_info_r2r(I_prc_to,irank)
          i_to   = mod(g_to-1,ADM_gall_1d) + 1
          j_to   = (g_to-i_to) / ADM_gall_1d + 1
          r_to   = RGNMNG_lp2r(l_to,p_to)

          write(IO_FID_LOG,'(13(I6))') ipos, i_from, j_from, r_from, g_from, l_from, p_from, &
                                             i_to  , j_to  , r_to  , g_to  , l_to  , p_to
       enddo
    enddo
  endif

  allocate( sendbuf_r2r(Send_size_nglobal*ADM_kall*COMM_varmax,Send_nmax_r2r) )
  allocate( recvbuf_r2r(Send_size_nglobal*ADM_kall*COMM_varmax,Recv_nmax_r2r) )

  return
end subroutine COMM_sortdest
```

This section writes the contents of `Copy_list_r2r`, `Recv_list_r2r`, `Send_list_r2r`, in the form of table, respectively. Note that `debug` is true in this kernel program. See subsection 2.9.3 for the example of this output.

Regarding to subroutine `COMM_sortdest_pl` and `COMM_sortdest_singular`, they do similar procedure with `COMM_sortdest`, omit here.

(4) `COMM_debugtest`

`COMM_debugtest` is to test actual halo-exchange communication routine, `COMM_data_transfer` and `COMM_var`.

The first section is as follows.

```
subroutine COMM_debugtest
  use mod_process, only: &
     PRC_MPIfinish
  use mod_adm, only: &
     ADM_prc_me,      &
     ADM_gall,        &
     ADM_gall_pl,     &
     ADM_kall,        &
     ADM_lall,        &
     ADM_lall_pl,     &
     ADM_lall_pl,     &
     ADM_gall_1d,     &
     ADM_gmin,        &
     ADM_gmax,        &
     ADM_kmin,        &
     ADM_kmax,        &
     ADM_vlink,       &
     ADM_have_sgp,    &
     RGNMNG_l2r,      &
     RGNMNG_vert_num, &
     I_N,             &
     I_S
  implicit none

```

```
   real(RP) :: var   (ADM_gall   ,ADM_kall,ADM_lall   ,4)
   real(RP) :: var_pl(ADM_gall_pl,ADM_kall,ADM_lall_pl,4)

   integer  :: i, j, k, l, ij, rgnid, prc
   !---------------------------------------------------------------------------

```

As the other subroutines in this section, this is also private in this module, and there are no arguments. `var` and `var_pl` are temporal array used in communication.

Main part are separated by two section. The fist section is as follows.

```
   write(IO_FID_LOG,*)
   write(IO_FID_LOG,*) '+++␣TEST␣start'

   var   (:,:,:,:) = -999.D0
   var_pl(:,:,:,:) = -999.D0

   do l = 1, ADM_lall
      rgnid = RGNMNG_l2r(l)
      prc   = ADM_prc_me

      do k = ADM_kmin, ADM_kmax
         do j = ADM_gmin, ADM_gmax
         do i = ADM_gmin, ADM_gmax
            ij = ADM_gall_1d * (j-1) + i

            var(ij,k,l,1) = real(prc,  kind=RP)
            var(ij,k,l,2) = real(rgnid,kind=RP)
            var(ij,k,l,3) = real(i,    kind=RP)
            var(ij,k,l,4) = real(j,    kind=RP)
         enddo
         enddo
      enddo

      if ( ADM_have_sgp(l) ) then
         do k = ADM_kmin, ADM_kmax
            var(1,k,l,:) = -1.D0
         enddo
      endif
   enddo

   do l = 1, ADM_lall_pl
      rgnid = l
      prc   = ADM_prc_me

      do k  = ADM_kmin, ADM_kmax
      do ij = 1, ADM_gall_pl
         var_pl(ij,k,l,1) = real(-prc,  kind=RP)
         var_pl(ij,k,l,2) = real(-rgnid,kind=RP)
         var_pl(ij,k,l,3) = real(-ij,   kind=RP)
         var_pl(ij,k,l,4) = real(-ij,   kind=RP)
      enddo
      enddo
   enddo

   write(IO_FID_LOG,*) "#####␣(prc,rgnid)␣#####"
   do l  = 1, ADM_lall
   do k = ADM_kmin, ADM_kmin
      write(IO_FID_LOG,*)
      write(IO_FID_LOG,'(A9)',advance='no') "␣␣␣␣␣␣␣␣|"
      do i = 1, ADM_gall_1d
         write(IO_FID_LOG,'(I9)',advance='no') i
      enddo
      write(IO_FID_LOG,*)

      do j = ADM_gall_1d, 1, -1
         write(IO_FID_LOG,'(I8,A1)',advance='no') j, "|"
         do i = 1, ADM_gall_1d
            ij = ADM_gall_1d * (j-1) + i
            write(IO_FID_LOG,'(A1,I3,A1,I3,A1)',advance='no') &
                 '(',int(var(ij,k,l,1)),',',int(var(ij,k,l,2)),')'
         enddo
         write(IO_FID_LOG,*)
      enddo
   enddo
   enddo

   do l  = 1, ADM_lall_pl
   do k = ADM_kmin, ADM_kmin
```

```fortran
   write(IO_FID_LOG,*)
   write(IO_FID_LOG,'(A9)',advance='no') "␣␣␣␣␣␣␣␣|"
   do i = 1, ADM_gall_pl
      write(IO_FID_LOG,'(I9)',advance='no') i
   enddo
   write(IO_FID_LOG,*)

   write(IO_FID_LOG,'(I8,A1)',advance='no') j, "|"
   do ij = 1, ADM_gall_pl
      write(IO_FID_LOG,'(A1,I3,A1,I3,A1)',advance='no') &
                 '(',int(var_pl(ij,k,l,1)),',',int(var_pl(ij,k,l,2)),')'
   enddo
   write(IO_FID_LOG,*)
enddo
enddo

write(IO_FID_LOG,*) "#####␣(i,j)␣#####"
do l  = 1, ADM_lall
do k = ADM_kmin, ADM_kmin
   write(IO_FID_LOG,*)
   write(IO_FID_LOG,'(A9)',advance='no') "␣␣␣␣␣␣␣␣|"
   do i = 1, ADM_gall_1d
      write(IO_FID_LOG,'(I9)',advance='no') i
   enddo
   write(IO_FID_LOG,*)

   do j = ADM_gall_1d, 1, -1
      write(IO_FID_LOG,'(I8,A1)',advance='no') j, "|"
      do i = 1, ADM_gall_1d
         ij = ADM_gall_1d * (j-1) + i
         write(IO_FID_LOG,'(A1,I3,A1,I3,A1)',advance='no') &
                    '(',int(var(ij,k,l,3)),',',int(var(ij,k,l,4)),')'
      enddo
      write(IO_FID_LOG,*)
   enddo
enddo
enddo

do l  = 1, ADM_lall_pl
do k = ADM_kmin, ADM_kmin
   write(IO_FID_LOG,*)
   write(IO_FID_LOG,'(A9)',advance='no') "␣␣␣␣␣␣␣␣|"
   do i = 1, ADM_gall_pl
      write(IO_FID_LOG,'(I9)',advance='no') i
   enddo
   write(IO_FID_LOG,*)

   write(IO_FID_LOG,'(I8,A1)',advance='no') j, "|"
   do ij = 1, ADM_gall_pl
      write(IO_FID_LOG,'(A1,I3,A1,I3,A1)',advance='no') &
                 '(',int(var_pl(ij,k,l,3)),',',int(var_pl(ij,k,l,4)),')'
   enddo
   write(IO_FID_LOG,*)
enddo
enddo



write(IO_FID_LOG,*)
write(IO_FID_LOG,*) '+++␣Communication␣start'

call COMM_data_transfer( var(:,:,:,:), var_pl(:,:,:,:) )

write(IO_FID_LOG,*) "#####␣(prc,rgnid)␣#####"
do l  = 1, ADM_lall
do k = ADM_kmin, ADM_kmin
   write(IO_FID_LOG,*)
   write(IO_FID_LOG,'(A9)',advance='no') "␣␣␣␣␣␣␣␣|"
   do i = 1, ADM_gall_1d
      write(IO_FID_LOG,'(I9)',advance='no') i
   enddo
   write(IO_FID_LOG,*)

   do j = ADM_gall_1d, 1, -1
      write(IO_FID_LOG,'(I8,A1)',advance='no') j, "|"
      do i = 1, ADM_gall_1d
         ij = ADM_gall_1d * (j-1) + i
         write(IO_FID_LOG,'(A1,I3,A1,I3,A1)',advance='no') &
                    '(',int(var(ij,k,l,1)),',',int(var(ij,k,l,2)),')'
      enddo
      write(IO_FID_LOG,*)
   enddo
```

```
  enddo
  enddo

  do l  = 1, ADM_lall_pl
  do k = ADM_kmin, ADM_kmin
     write(IO_FID_LOG,*)
     write(IO_FID_LOG,'(A9)',advance='no') "        |"
     do i = 1, ADM_gall_pl
        write(IO_FID_LOG,'(I9)',advance='no') i
     enddo
     write(IO_FID_LOG,*)

     write(IO_FID_LOG,'(I8,A1)',advance='no') j, "|"
     do ij = 1, ADM_gall_pl
        write(IO_FID_LOG,'(A1,I3,A1,I3,A1)',advance='no') &
                   '(',int(var_pl(ij,k,l,1)),',',int(var_pl(ij,k,l,2)),')'
     enddo
     write(IO_FID_LOG,*)
  enddo
  enddo

  write(IO_FID_LOG,*) "##### (i,j) #####"
  do l  = 1, ADM_lall
  do k = ADM_kmin, ADM_kmin
     write(IO_FID_LOG,*)
     write(IO_FID_LOG,'(A9)',advance='no') "        |"
     do i = 1, ADM_gall_1d
        write(IO_FID_LOG,'(I9)',advance='no') i
     enddo
     write(IO_FID_LOG,*)

     do j = ADM_gall_1d, 1, -1
        write(IO_FID_LOG,'(I8,A1)',advance='no') j, "|"
        do i = 1, ADM_gall_1d
           ij = ADM_gall_1d * (j-1) + i
           write(IO_FID_LOG,'(A1,I3,A1,I3,A1)',advance='no') &
                      '(',int(var(ij,k,l,3)),',',int(var(ij,k,l,4)),')'
        enddo
        write(IO_FID_LOG,*)
     enddo
  enddo
  enddo

  do l  = 1, ADM_lall_pl
  do k = ADM_kmin, ADM_kmin
     write(IO_FID_LOG,*)
     write(IO_FID_LOG,'(A9)',advance='no') "        |"
     do i = 1, ADM_gall_pl
        write(IO_FID_LOG,'(I9)',advance='no') i
     enddo
     write(IO_FID_LOG,*)

     write(IO_FID_LOG,'(I8,A1)',advance='no') j, "|"
     do ij = 1, ADM_gall_pl
        write(IO_FID_LOG,'(A1,I3,A1,I3,A1)',advance='no') &
                   '(',int(var_pl(ij,k,l,3)),',',int(var_pl(ij,k,l,4)),')'
     enddo
     write(IO_FID_LOG,*)
  enddo
  enddo
```

This section does the test of `COMM_data_transfer`. As test data, process number `prc`, region number `rgnid`, and the grid index `i`, `j` are set to `var` and `var_pl`. The values of these arrays of before and after the calling `COMM_data_transfer` are written to the log (standard out).

The latter section is as follows.

```
  do l = 1, ADM_lall
     rgnid = RGNMNG_l2r(l)
     prc   = ADM_prc_me

     if ( RGNMNG_vert_num(I_N,rgnid) == ADM_vlink ) then
        do k = ADM_kmin, ADM_kmax
           j  = ADM_gmax+1
           i  = ADM_gmin
           ij = ADM_gall_1d * (j-1) + i
```

```
            var(ij,k,l,1) = real(prc,  kind=RP)
            var(ij,k,l,2) = real(rgnid,kind=RP)
            var(ij,k,l,3) = real(i,    kind=RP)
            var(ij,k,l,4) = real(j,    kind=RP)
         enddo
      endif

      if ( RGNMNG_vert_num(I_S,rgnid) == ADM_vlink ) then
         do k = ADM_kmin, ADM_kmax
            j  = ADM_gmin
            i  = ADM_gmax+1
            ij = ADM_gall_1d * (j-1) + i

            var(ij,k,l,1) = real(prc,  kind=RP)
            var(ij,k,l,2) = real(rgnid,kind=RP)
            var(ij,k,l,3) = real(i,    kind=RP)
            var(ij,k,l,4) = real(j,    kind=RP)
         enddo
      endif

   enddo

   write(IO_FID_LOG,*)
   write(IO_FID_LOG,*) '+++␣pole␣fill␣start'

   call COMM_var( var(:,:,:,:), var_pl(:,:,:,:), ADM_kall, 4 )

   write(IO_FID_LOG,*) "#####␣(prc,rgnid)␣#####"
   do l  = 1, ADM_lall
   do k = ADM_kmin, ADM_kmin
      write(IO_FID_LOG,*)
      write(IO_FID_LOG,'(A9)',advance='no') "␣␣␣␣␣␣␣␣|"
      do i = 1, ADM_gall_1d
         write(IO_FID_LOG,'(I9)',advance='no') i
      enddo
      write(IO_FID_LOG,*)

      do j = ADM_gall_1d, 1, -1
         write(IO_FID_LOG,'(I8,A1)',advance='no') j, "|"
         do i = 1, ADM_gall_1d
            ij = ADM_gall_1d * (j-1) + i
            write(IO_FID_LOG,'(A1,I3,A1,I3,A1)',advance='no') &
                       '(',int(var(ij,k,l,1)),',',int(var(ij,k,l,2)),')'
         enddo
         write(IO_FID_LOG,*)
      enddo
   enddo
   enddo

   do l  = 1, ADM_lall_pl
   do k = ADM_kmin, ADM_kmin
      write(IO_FID_LOG,*)
      write(IO_FID_LOG,'(A9)',advance='no') "␣␣␣␣␣␣␣␣|"
      do i = 1, ADM_gall_pl
         write(IO_FID_LOG,'(I9)',advance='no') i
      enddo
      write(IO_FID_LOG,*)

      write(IO_FID_LOG,'(I8,A1)',advance='no') j, "|"
      do ij = 1, ADM_gall_pl
         write(IO_FID_LOG,'(A1,I3,A1,I3,A1)',advance='no') &
                    '(',int(var_pl(ij,k,l,1)),',',int(var_pl(ij,k,l,2)),')'
      enddo
      write(IO_FID_LOG,*)
   enddo
   enddo

   write(IO_FID_LOG,*) "#####␣(i,j)␣#####"
   do l  = 1, ADM_lall
   do k = ADM_kmin, ADM_kmin
      write(IO_FID_LOG,*)
      write(IO_FID_LOG,'(A9)',advance='no') "␣␣␣␣␣␣␣␣|"
      do i = 1, ADM_gall_1d
         write(IO_FID_LOG,'(I9)',advance='no') i
      enddo
      write(IO_FID_LOG,*)

      do j = ADM_gall_1d, 1, -1
         write(IO_FID_LOG,'(I8,A1)',advance='no') j, "|"
         do i = 1, ADM_gall_1d
            ij = ADM_gall_1d * (j-1) + i
```

```
            write(IO_FID_LOG,'(A1,I3,A1,I3,A1)',advance='no') &
                       '(',int(var(ij,k,l,3)),',',int(var(ij,k,l,4)),')'
         enddo
         write(IO_FID_LOG,*)
      enddo
   enddo
   enddo

   do l  = 1, ADM_lall_pl
   do k = ADM_kmin, ADM_kmin
      write(IO_FID_LOG,*)
      write(IO_FID_LOG,'(A9)',advance='no') "        |"
      do i = 1, ADM_gall_pl
         write(IO_FID_LOG,'(I9)',advance='no') i
      enddo
      write(IO_FID_LOG,*)

      write(IO_FID_LOG,'(I8,A1)',advance='no') j, "|"
      do ij = 1, ADM_gall_pl
         write(IO_FID_LOG,'(A1,I3,A1,I3,A1)',advance='no') &
                    '(',int(var_pl(ij,k,l,3)),',',int(var_pl(ij,k,l,4)),')'
      enddo
      write(IO_FID_LOG,*)
   enddo
   enddo

   return
end subroutine COMM_debugtest
```

This section does the test of `COMM_var`. The procedure is the same with the case of `COMM_data_transfer`, using `prc`, `rgnid`, `i` and `j` as a test data, writes out these values the before and after calling this subroutine.

See subsection 2.9.3 for example.

2.9.3 Input data and result

This kernel program doesn't need any input data, except configuring namelist file `communication.cnf`.

Several configuration is prepared in `data` directory. The default one is as follows.

```
################################################################################

&ADMPARAM
    glevel = 4,
    rlevel = 0,
    vlayer = 20,
    debug  = .true.,
/

&COMMPARAM
    debug    = .true.,
    testonly = .true.,
/

################################################################################
```

Script files in `run` directory executes this kernel program with two MPI processes. Each process outputs logfile named `msg.pe000000` and `msg.pe000001`.

These files are so long and skip here. There are example output files in `reference` directory. These outputs must be the same except white space changes.

2.9.4 Sample of perfomance result

Here's an example of the performance result part of the log output. Below is an example executed with the machine environment described in subsection 2.1.7. Note that in this program kernel part is iterated one time.

```
 *** Computational Time Report
 *** Rap level is             2
 *** ID=001 : MAIN_Kernel_ALL                  T=     0.020 N=        1
 *** ID=002 : MAIN_COMM_data_transfer          T=     0.001 N=        2
 *** ID=003 : MAIN_COMM_var                    T=     0.000 N=        1
```

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IcoAtmosBenchmark NICAM kernels

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